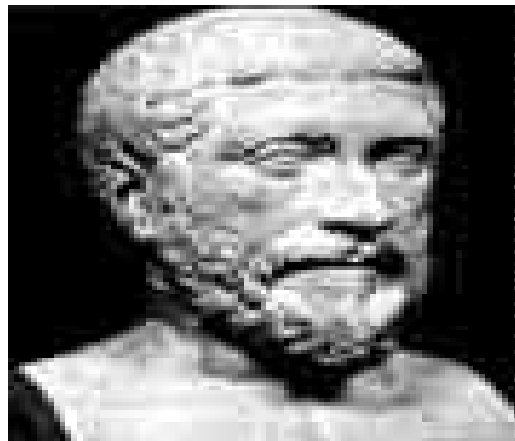
Program Pasca Sarjana (PPs)

MODUL

FILSAFAT ILMU

Untuk:
Program S2 Pendidikan Matematika
dan Pendidikan Sain Bilingual

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KEMENTERIAN PENDIDIKAN NASIONAL

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Preface

The author has been lucky to have a chance to write this Modul. This Modul has a purpose to facilitate students of S2 Study Program of Mathematics Education and Sain in Program Pasca Sarjana (PPs), Yogyakarta State University, to learn intensivelu and extensively about the Philosophy of Science (Filsafat Ilmu).

The author wish to express thank to whoever support this publication. Specifically, the author would like to give special thank to :

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Hope that by publication of this Modul, teaching learning program at PPs Yogyakarta State University will be carried out smoothly.

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Author

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INTRODUCTION

Among sciences, *mathematics* has a unique relation to philosophy. Many philosophers have taken *mathematics* to be the paradigm of knowledge, and the reasoning employed in some mathematical proofs are often regarded as rational thought, however mathematics is also a rich source of philosophical problems which have been at the centre of *epistemology* and *metaphysics*. Since antiquity[1], philosophers have envied their *ideas* as the *model of mathematical perfection* because of the clarity of its concepts and the certainty of its conclusions.

Many efforts should be done if we strive to elaborate philosophy and foundation of mathematics. Philosophy of mathematics covers the discussion of ontology of mathematics, epistemology of mathematics, mathematical truth, and mathematical objectivity. While the foundation of mathematics engages with the discussion of ontological foundation, epistemological foundation which covers the schools of philosophy such as *platonism, logicism, intuitionism, formalism,* and *structuralism.*

Philosophy of mathematics, as it was elaborated by Ross D.S. (2003), is a philosophical study of the concepts and methods of mathematics. According to him, *philosophy of mathematics* is concerned with the *nature of numbers, geometric objects,* and other *mathematical concepts;* it is concerned with their cognitive origins and with their application to reality. Further, it addresses the validation of methods of mathematical inference. In particular, it deals with the logical problems associated with mathematical infinitude. Meanwhile, Hersh R. (1997) thinks that *Philosophy of mathematics* should articulate with epistemology and philosophy of science; but virtually all writers on philosophy of mathematics treat *it as an encapsulated entity, isolated, timeless, a-historical, inhuman, connected to nothing else in the intellectual or material realms.*

Philip Kitcher[2] indicates that the *philosophy of mathematics* is generally supposed to begin with Frege due to he transformed the issues constituting philosophy of mathematics. Before Frege, the philosophy of mathematics was only "*prehistory*." To understand Frege, we must see him as a *Kantian.* To understand Kant we must see his response to Newton, Leibniz, and Hume. Those three philosophers go back to Descartes and through him they

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[1] -----, 2003, "*Mathematics, mind, ontology and the origins of number*". Retreived 2004

<http://website.lineone.net/~kwelos/>

[2] *Ibid*.

back to Plato. Platos is a *Pythagorean.* The thread from Pythagorean to Hilbert and Godel is unbroken. A connected story from Pythagoras to the present is where the foundation came from. Although we can connect the thread of the foundation of mathematics from the earlier to the present, we found that some philosophers have various interpretation on *the nature of mathematics* and its *epistemological foundation.*

While Hilary Putnam in Hersh, R. (1997), a *contemporary* philosopher of mathematics, argues that the subject matter of mathematics is the physical world and not its actualities, but its potentialities. According to him, to exist in mathematics means to exist potentially in the physical world. This interpretation is attractive, because in facts mathematics is meaningful, however, it is unacceptable, because it tries to explain *the clear* by *the obscure*. On the other hand, Shapiro in Linebo, O states that there are two different orientations of relation between *mathematical practice* and *philosophical theorizing*; *first,* we need a philosophical account of what mathematics is about, only then can we determine what qualifies as correct mathematical reasoning; the other orientation of mathematics is an autonomous science so it doesn't need to borrow its authority from other disciplines.

On the *second* view[3], philosophers have no right to legislate mathematical practice but must always accept mathematicians' own judgment. Shapiro insists that philosophy must also interpret and make sense of mathematical practice, and that this may give rise to criticism of oral practice; however, he concedes that this criticism would have to be *internal* to mathematical practice and take 'as *data* that most of *contemporary* mathematics is correct'. Shapiro confesses whether mathematicians should really be regarded as endorsing philosophical theorizing will depend on what is meant by '*accurately represents the semantic form of mathematical language*'. If the notions of semantic form and truth employed in philosophical theorizing are understood in a *deflationary* way, it is hard to see how philosophical theorizing can go beyond mathematicians' claim that the realist principles are literally true. On the other hand, Stefanik, 1994, argues whether the philosophy of mathematics most fruitfully pursued as a philosophical investigation into the nature of numbers as abstract entities existing in a platonic realm inaccessible by means of our standard perceptual capacities, or the study of the practices and activities of mathematicians with special emphasis on the nature of the fundamental *objects* that are the concern of actual mathematical research.

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[3] Shapiro in Linnebo, Ø., 2003, "*Review of Stewart Shapiro, Philosophy of Mathematics: Structure and Ontology"*, Retrieved 2004 < *http://www.oystein.linnebo@filosofi.uio.no*>

CHAPTER ONE
PHILOSOPHY OF MATHEMATICS AND SCIENCE

Kompetensi Dasar Mahasiswa:

Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap persoalan-persoalan pokok dalam Pengembangan Filsafat Matematika dan Sain

Materi Pokok :

Persoalan-persoalan Pokok dalam Pengembangan Filsafat Matematika dan Sain

Strategi Perkuliahan:

Ekspositori, diskusi, presentasi, refleksi

Sumber Bahan:

Pilih yang sesuai

Contemporary, our view of *mathematical knowledge*[4] should fit our view of knowledge in general; if we write philosophy of mathematics, we aren't expected simultaneously to write philosophy of science and general epistemology. Hersh R. suggests that to write philosophy of mathematics alone is daunting enough; but to be adequate, it needs a connection with epistemology and philosophy of science. *Philosophy of mathematics* can be tested against some mathematical practices: *research, application, teaching, history,* and *computing.* In more universal atmosphere, Posy, C., (1992) states that philosophy of mathematics should involve the *epistemology, ontology,* and *methodology of mathematics.* Accordingly[5], certain aspects unique to mathematics cause its philosophy to be of particular interest: 1) *abstraction* - math involves abstract concepts ; 2) *application* - math is used by other sciences like Physics ; 3) *infinity* - peculiar notion specifically to pure math, yet a central concept to applied calculations . Specific events[6] caused the evolution of mathematical views in an attempt to eliminate cracks in the

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[4] Hersh, R., 1997, *"What is Mathematics, Really?"*, London: Jonathan Cape, p. 236

[5] Posy, C., 1992, "Philosophy of Mathematics", Retrieved 2004 <*http://www.cs.washington.edu/ homes/ gjb.doc/philmath.htm*>

[6] *Ibid*.

foundation of mathematics. The most important of these was the discovery of inconsistencies, or paradoxes, in the foundations of mathematics; this represents the starting point of the modern philosophy of mathematics.

1. Ontology of Mathematics

Ross, D.S. (2003) states that there are some ontological questions in the *Philosophy of Mathematics:*

> What is the origin of mathematical objects? In what way do mathematical objects exist? Have they always been present as 'Platonic' abstractions, or do they require a mind to bring them into existence? Can mathematical objects exist in the absence of matter or things to count?.

Since the beginning of *Western philosophy*[7], there are important philosophical problems: *Do numbers and other mathematical entities exist independently on human cognition? If they exist dependently on human cognition then how do we explain the extraordinary applicability of mathematics to science and practical affairs?* If they exist independently on human cognition then what kind of things are they and how can we know about them? And what is the relationship between *mathematics* and *logic*? The first question is a metaphysical question with close affinities to questions about the existence of other entities such as *universals, properties* and *values*. According to many philosophers, if such entities exist then they do beyond the space and time, and they lack of causal powers. They are often termed abstract as opposed to concrete entities. If we accept the existence of abstract mathematical objects then an adequate *epistemology of mathematics* must explain how we can know them; of course, proofs seem to be the main source of justification for mathematical propositions but proofs depend on *axioms* and so the question of how we can know the truth of the *axioms* remains.

It is advocated especially by Stuart Mill J. in Hempel C.G. (2001) that mathematics itself is *an empirical science* which differs from the other branches such as astronomy, physics, chemistry, etc., mainly in two respects: its subject matter is more general than that of any other field of scientific research, and its propositions have been tested and confirmed to a greater extent than those of even the most firmly established sections of astronomy or

[7] *Ibid.*

physics. According to Stuart Mill J., the degree to which the *laws of mathematics* have been born out by the past experiences of mankind is so unjustifiable that we have come to think of *mathematical theorems* as qualitatively different from the well confirmed hypotheses or theories of other branches of science in which we consider them as certain, while other theories are

thought of as at best as very probable or very highly confirmed and of course this view is open to serious objections.

While Hempel C.G. himself acknowledges that, from *empirical hypothesis*, it is possible to derive predictions to the effect that under certain specified conditions, certain specified observable phenomena will occur; the actual occurrence of these phenomena constitutes confirming evidence. It[8] was concluded that an *empirical hypothesis* is theoretically *un-confirmable* that is possible to indicate what kind of evidence would disconfirm the *hypothesis*; if this is actually an *empirical generalization of past experiences,* then it must be possible to state what kind of evidence would oblige us to concede the hypothesis not generally true after all. The *mathematical propositions* are true simply by virtue of definitions or of similar stipulations which determine the meaning of the key terms involved. Soehakso, RMJT, guides that mathematics validation naturally *requires no empirical evidence;* they can be shown to be true by a mere analysis of the meaning attached to the terms which occur in. The *exactness and rigor* of mathematics[9] means that the understanding of mathematics follows the logical development of important peculiar mathematical methods, and of course is acquainted with major results especially in "*foundations*".

The *validity of mathematics*, as it was stated by Hempel C.G., rests neither on its alleged *self-evidential* character nor on any empirical basis, but it also derives from the stipulations which determine the meaning of the mathematical concepts, and that the propositions of mathematics are therefore essentially "*true by definition.*" The *rigorous*[10] development of a mathematical theory proceeds not simply from a set of definitions but rather from *a set of non-definitional propositions* which are not proved within the theory.

[8] In Hempel, C.G., 2001, "*On the Nature of Mathematical Truth"*, Retrieved 2004 <*http://www. ltn.lv/~podniek/gt.htm*>

[9] Soehakso, RMJT, 1989, "*Some Thought on Philosophy and Mathematics"*, Yogyakarta: Regional Conference South East Asian Mathematical Society, p.3

[10] In Hempel, C.G., 2001, "*On the Nature of Mathematical Truth"*, Retrieved 2004 <*http://www. ltn.lv/~podniek/gt.htm*>

Hempel states that there are the *postulate*s or *axioms* of the theory and formulated in terms of certain basic or primitive concepts for which no definitions are provided within the theory. The *postulate*s themselves represent *"implicit definitions"* of the primitive terms while the *postulate*s do limit, in a specific sense, the meanings that can possibly be ascribed to the primitives, any *self-consistent postulate* system admits.[11]

Once the *primitive terms* and the *postulate*s[12] have been laid down the entire theory is completely determined. Hence, every term of the mathematical theory is *definable* in terms of the primitives, and every proposition of the theory is logically deducible from the *postulate*s. Hempel adds that to be entirely precise, it is *necessary* to specify the principles of logic used in the proof of the propositions; these principles can be stated quite explicitly and fall into primitive sentences or *postulate*s of logic. Accordingly, any fact that we can derive from the *axioms* needs not be an axiom; anything that we cannot derive from the *axioms* and for which we also cannot derive the negation might reasonably added as an axiom. Hempel concludes that by combining the analyses of the aspects of the *Peano system,* the thesis of *logicism* was accepted that Mathematics is a branch of logic due to all the concepts of mathematics i.e. *arithmetic, algebra,* and *analysis* can be defined in terms of four concepts of pure logic and all the *theorems* of mathematics can be deduced from those definitions by means of the principles of logic.

2. Epistemology of Mathematics

Mathematics[13] is about the structure of immediate experience and the potentially *infinite* progression of sequences of such experiences; it involves the creation of truth which has an objective meaning in which its statements that cannot be interpreted as questions about events all of which will occur in a potentially *infinite* deterministic universe are neither true nor false in any absolute sense. They may be useful properties[14] that are either true or false relative to a particular *formal system*. Hempel C.G. (2001) thought that the truths of mathematics, in contradistinction to the hypotheses of empirical science, require neither factual evidence nor any other justification because they are *self-evident*. However, the

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[11] *Ibid*

[12] *Ibid*.

[13] --------, 2004, "A philosophy of mathematical truth", Mountain Math Software, Retrieved 2004 <webmaster@mtnmath.com.>

[14] *Ibid.*

existence of *mathematical conjectures*[15] shows that not all *mathematical truths* can be *self-evident* and even if *self-evidence* were attributed only to the basic *postulate*s of mathematics.

Hempel C.G. claims that *mathematical judgments* as to what may be considered as *self-evident* are subjective that is they may vary from person to person and certainly cannot constitute an adequate basis for decisions as to the *objective validity* of mathematical propositions. While Shapiro[16] perceives that we learn perceptually that individual objects and systems of objects display a variety of patterns and we need to know more about the epistemology of the crucial step from the perspective of *places-as-offices*, which has no abstract commitments, to that of *places-as-objects*, which *is* thus committed. He views that *mathematical objects* can be introduced by abstraction on an equivalence relation over some prior class of entities. Shapiro[17] invokes an epistemological counterpart that, by laying down an implicit definition and convincing ourselves of its coherence, we successfully refer to the structure it defines.

a. Mathematical Truth

It[18] is usually thought that *mathematical truths* are *necessary truths.* Two main views are possible i.e. either they are known by *reason* or they are known by inference from *sensory experience.* The former *rationalist* view is adopted by Descartes and Leibniz who also thought that *mathematical concepts are innate*; while Locke and Hume agreed that *mathematical truths* are known by reason but they thought all mathematical concepts were derived by abstraction from experience. Mill[19] was a complete empiricist about mathematics and held that mathematical concepts are derived from experience and also that *mathematical truths* are really *inductive generalizations from experience.* Weir A. theorizes that one obvious problem for *neo-formalism* is its apparent conflict with *Gödel's first incompleteness* result showing that not all *mathematical truths* are provable, under *a certain*

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[15] Hempel, C.G., 2001, *"On the Nature of Mathematical Truth*", Retrieved 2004 <http://www.ltn.lv/ ~podniek/gt.htm>
[16] Shapiro in Linnebo, Ø., 2003, "*Review of Stewart Shapiro, Philosophy of Mathematics: Structure and Ontology"*, Retrieved 2004 < *http://www.oystein.linnebo@filosofi.uio.no*>
[17] In Linnebo, Ø., 2003, *"Review of Stewart Shapiro, Philosophy of Mathematics: Structure and Ontology",* Retrieved 2004 < *http://www.oystein.linnebo@filosofi.uio.no*>
[18] Philosophy of Mathematics, Retrieved 2004 <http://Googlesearch.*philosophy.math*>
[19] *Ibid*

conception of provability. Even though the *neo-formalist*[20] makes no synonymy claim between '*sixty eight and fifty seven equals one hundred and twenty five'* and "*68+57=125*" is *provable'*, this result seems to rule out any tight equivalence between truth and proof of the sort envisaged. According to Weir A., we need a distinction between *legitimate* and *illicit transformations*, if *neo-formalism* is to avoid the consequence that in mathematics there is no distinction between *truth* and *falsity*. It[21] cannot be that a string is provable if derivable in the one true logic from some consistent set of *axioms* or other; even if there is only one true logic it would still follow that any logically consistent sentence i.e. *a mathematical truth.*

The *neo-formalist,* as it was notified by Weir A., perceives that *provability* in a practice means derivable using only inference rules which are in some sense analytic and *constitutive* of the meaning of our logical and mathematical operators. There are the responses to the *neo-formalist,* that no rule can be *meaning-constitutive* if it is *trivial.* The inconsistency and indeed triviality of *'classical' naïve set theory*[22] is a product of three things: *the classical operational rules, the classical structural rules and the naïve rules or axioms.* The *neo-formalist*[23] agrees with the *strict finitist* that the only objects with a title to being called *mathematical* which exist in reality are the presumably finite number of concrete mathematical utterances; some of these utterances, however, are used to assert that *infinite*ly many objects- *numbers, sets, strings of expressions, abstract proofs, etc.-* exist.

Mathematical truth[24] is thus linked with *provability* in *formal* calculi and in such a way as to be perfectly compatible with the claim that all that exists in *mind-dependent reality* are concrete objects together with their physical properties. Field H. observes that the d*eterminacy of mathematical statements* is primarily dependent on the precision we can give to the *semantics of the language* in which they are expressed. If [25] we are dealing with mathematics expressed in *first order logic*, then the *semantics of the logic* itself *are pretty well nailed dow*n and if the theory under consideration concerns a unique structure up to *isomorphism* then we know that each closed sentence will have a definite *truth value*

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[20] Weir A., 2004, "*A Neo-Formalist Approach to Mathematical Truth",* Retrieved 2004 <*http://en. wikipedia.org/wiki/GNU_FDL*>
[21] *Ibid.*
[22] *Ibid.*
[23] *Ibid.*
[24] *Ibid.*
[25] Field, H., 1999, "*Which Undecidable Mathematical Sentences Have Determinate Truth Values?"*, RJB, Retrieved 2004 <*http://www.rbjones.com/rbjpub/rbj.htm*>

under that interpretation, and there will only be indeterminacy if there is some substantive ambiguity about what this unique intended interpretation is.

Field H. [26], claims that even if as in the case of *arithmetic,* there can be no complete *recursive axiomatization* of the theory, which will normally be the case where there is a unique intended interpretation. On the other hand, Oddie G. says:

> While mathematical truth is the aim of inquiry, some falsehoods seem to realize this aim better than others; some truths better realize the aim than other truths and perhaps even some falsehoods realize the aim better than some truths do. The dichotomy of the class of propositions into truths and falsehoods should thus be supplemented with a more fine-grained ordering -- one which classifies propositions according to their closeness to the truth, their degree of truth-likeness or verisimilitude. The problem of truth-likeness is to give an adequate account of the concept and to explore its logical properties and its applications to epistemology and methodology. [27]

Popper[28] refers to Hume's notion that we not only that we can not verify an interesting theory, we can not even render it more probable. There[29] is an *asymmetry* between *verification* and *falsification* and while no finite amount of data can verify or probability an interesting scientific theory, they can *falsify the theory*. Popper[30] indicates that it is the *falsifiability* of a theory which makes it scientific; and it implied that the only kind of progress an inquiry can make consists in *falsification of theories.* Popper states that if some false hypotheses are closer to the truth than others, if verisimilitude admits of degrees, then the history of inquiry may turn out to be one of steady progress towards the goal of truth. It may be reasonable, on the basis of the evidence, to conjecture that our theories are indeed making such progress even though it would be unreasonable to conjecture that they are true *simpliciter.*[31]

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[26] *Ibid*.
[27] Oddie, G., 2001, *"Truthlikeness",* Stanford Encyclopedia of Philosophy, Paideia, Philosophy of Mathematics. Retrieved 2004 <*elaine@philo.mcgill.ca*>
[28] Popper in Oddie, G., 2001, *"Truthlikeness",* Stanford Encyclopedia of Philosophy, Paideia, Philosophy of Mathematics. Retrieved 2004 <*elaine@philo.mcgill.ca*>
[29] Oddie, G., 2001, *"Truthlikeness",*, Stanford Encyclopedia of Philosophy, Paideia, Philosophy of Mathematics. Retrieved 2004 <*elaine@philo.mcgill.ca*>
[30] Popper in Oddie, G., 2001, *"Truthlikeness",* Stanford Encyclopedia of Philosophy, Paideia, Philosophy of Mathematics. Retrieved 2004 <*elaine@philo.mcgill.ca*>
[31] *Ibid.*

Again, Oddie G. convicts that the quest for theories with high probability must be quite *wrong-headed*, while we want inquiry to yield true propositions, in which not any old truths will do. A *tautology*[32] is a truth, and as certain as anything can be, but it is never the answer to any interesting inquiry outside mathematics and logic. What we want are deep truths, truths which capture more rather than less, of the whole truth. Even more important, there is a difference between being true and being the truth. The truth, of course, has the property of being true, but not every proposition that is true is the truth in the sense required by the aim of inquiry. The truth of a matter at which an inquiry aims has to be the complete, true answer. Oddie G. illustrates the following:

> The world induces a partition of sentences of L into those that are true and those that are false. The set of all true sentences is thus a complete true account of the world, as far as that investigation goes and it is aptly called the Truth, T. T is the target of the investigation couched in L and it is the theory that we are seeking, and, if truthlikeness is to make sense, theories other than T, even false theories, come more or less close to capturing T. T, the Truth, is a theory only in the technical Tarskian sense, not in the ordinary everyday sense of that term. It is a set of sentences closed under the consequence relation: a consequence of some sentences in the set is also a sentence in the set. T may not be finitely axiomatisable, or even axiomatisable at all. The language involves elementary arithmetic it follows that T won't be axiomatisable; however, it is a perfectly good set of sentences all the same. [33]

If a *mathematical truth*[34] is taken to be represented by *a unique model*, or complete possible world, then we end up with results very close to Popper's truth content account. In particular, *false propositions* are closer to the truth the stronger they are; however, if we take the *structuralist approach* then we will take the relevant states of affairs to be *"small"* states of affairs viz. chunks of the world rather than the entire world and then the possibility of more fine-grained distinctions between theories opens up. Further, Oddie G. (2001) claims that a theory can be false in very many different ways; the degree of verisimilitude of a true theory may also vary according to where the truth lies. One does not necessarily

[32] Oddie, G., 2001, "*Truthlikeness*", Stanford Encyclopedia of Philosophy, Paideia, Philosophy of Mathematics. Retrieved 2004 <*elaine@philo.mcgill.ca*>
[33] *Ibid*.
[34] *Ibid*.

make a step toward the truth by reducing the content of a false proposition. Nor does one necessarily make a step toward the truth by increasing the content of a false theory.

b. Mathematical Objectivity

Rand A. et al theorizes that *objectivism* recognizes a deeper connection between *mathematics* and *philosophy* than advocates of other philosophies have imagined. The process of *concept-formation*[35] involves the grasp of quantitative relationships among units and the omission of their specific measurements. They view that it thus places mathematics at the core of human knowledge as a crucial element of the *process of abstraction.* While, Tait W.W (1983) indicates that the question of objectivity in mathematics concerns, not primarily the existence of objects, but the objectivity of mathematical discourse. *Objectivity* in mathematics[36] is established when meaning has been specified for *mathematical propositions*, including *existential propositions*. This obviously resonates with Frege's so-called *context principle,* although Frege seems to have rejected the general view point of Cantor and, more explicitly, Hilbert towards mathematical existence.

Tait W.W points that the question of objective existence and truth concern the *axiomatic method* as it posit for '*concrete' mathematics*, i.e. *axioms of logic* and the *theory of finite* and *transfinite numbers* and the cumulative hierarchy of sets over them; in which we can reason about *arbitrary groups, spaces* and the like, and can construct examples of them. The *axiomatic method*[37] seems to run into difficulties. If *meaning* and *truth* are to be determined by what is *deducible from the axioms*, then we ought to require at least that the *axioms* be *consistent,* since otherwise the distinctions true/false and existence/non-existence collapse.

Tait W. W claims that there is *an external criterion* of mathematical existence and truth and that numbers, functions, sets, etc., satisfy it, is often called *'Platonism'*; but Plato deserves a better fate. Wittgenstein, at least in analogous cases, called it '*realism*'; but he wants to save this term for the view that we can truthfully assert the existence of numbers and the like without explaining the assertion away as saying something else. From the

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[35] Rand, A, Binswanger, H., Peikoff, P.,1990, "*Introduction to Objectivist Epistemology*", Retrieved 2004 *<http://www.literature-reviews.com/ Objectivism_ The_Philosophy _of_Ayn_Rand_Library_ Volume _6_0452011019.html>*

[36] Tait, W.W., 1983, "*Beyond the axioms: The question of objectivity in mathematics*", Retrieved 2004 *<http://home.uchicago.edu/~wwtx/objectivity.pdf.>*

[37] *Ibid*.

realist perspective[38], *what is objective* are the grounds for *judging truth of mathematical propositions,* including existential ones, namely, derivability from the *axioms*. However, to hold that there is some ground beyond this, to which the *axioms* themselves are *accountable,* is to enter the domain of *super-realism,* where mathematics is again *speculative*; if the *axioms* are *accountable,* then they might be *false*.[39]

Tait W.W (1983) emphasizes that our mathematical knowledge is *objective* and *unchanging* because it's knowledge of objects external to us, independent of us, which are indeed changeless. From the view of *realists*[40], our mathematical knowledge, in the sense of what is known, is *objective,* in that the criterion for truth, namely provability, is public. He further states that the criterion depends on the fact that we agree about what counts as a correct application—*what counts as a correct proof.* Nevertheless, there is such agreement and the criterion is the same for all and is in no way subjective. As long as there remains an objective criterion for truth, namely provability from the *axioms*, it is inessential to this conception that there always remain some indeterminate propositions. However, there is a further challenge to *realism* which seems to cut deeper, because it challenges the idea that provability from the *axioms* is *objective.* [41]

3. Axiology of Mathematics

In the contemporary times, the mathematical backbone of its value has been extensively investigated and proven over the past ten years. According to Dr. Robert S. Hartman's, value is a phenomena or concept, and the value of anything is determined by the extent to which it meets the intent of its meaning. Hartman[42] indicated that the value of mathematics has four dimensions: *the value of its meaning, the value of its uniqueness, the value of its purpose,* and *the value of its function*. Further, he suggested that these four "Dimensions of Value" are always referred to as the following concepts: *intrinsic value, extrinsic value,* and *systemic value.* The bare intrinsic and inherent essence of mathematical object[43] is a greater, developed intensity of immediacy. Mathematical object

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[38] *Ibid*
[39] *Ibid*
[40] *Ibid*.
[41] *Ibid.*
[42] ………, 2002, Validity Studies Of The Hartman Profile Model, Ai, Axiometrics International, Inc. Retrieved 2007 <http://www.google.com>
[43] Dagobert D. Runes, 1942, Dictionary of Philosophy. Retrieved 2007 <http://www. ditext. com/ index.html>

is genuinely independent either of consciousness or of other things; something for itself. In and for itself belongs to the Absolute alone, mathematical object can be perceived as the developed result of its nature and as a system of internal relations in which it is independent of external relations.

The important value distinctions like the hierarchy of value of mathematics can be stated that in the simplest instances of combining *intrinsic valuation* with its *extrinsic value,* and *systemic value* of mathematical object. Here, nothing gets lost, the hierarchy of value is preserved within the domain of intrinsic valuation, and it makes good sense to say that our value of mathematics has more value than the others. To illustrate the schema[44], let us take to modify Moore's model of the hierarchy of the valuation of mathematical object as follows: "$IA_1 = EA_1 = SA_1 = A_2$". "I" stands for all intrinsic value of mathematical object A1, "E" stands for all extrinsic value of mathematical object A1, and "S" stands for all systemic value of mathematical object A1. This formula acknowledges[45] that each of the first three formulas results in a value increase, so we end up with "A_2" on the far right. This formula also says that in each case we end up with the same value increase, namely, "A_2," and that is precisely the value of mathematics.

By the above method, Moore constructed a value of mathematics that can be used to objectively measure our capacity to make good value decisions. Unlike all other assessments, this structure allows us to understand 'how' a person thinks and perceives, not what they think, but how they think. Mathematics valuation explains and measures the thinking that forms the foundation for, and leads to, behavior. What's even more important is that one strives to value mathematics as objective, independent of any one observer, and accurate regardless of race, religion, socioeconomic conditions, or nationality. Valuation of mathematics are then applied to the rankings to provide numeric reference to the results. The individual uniqueness[46] of every human being is evidenced by the fact that there are 6.4 quadrillion possible permutations of these rankings.

In term of the practical sense, one way to value mathematics to the real world might be to introduce additional constants in the first order set theory to refer to things in the real world, and then to instantiate general mathematical theories to apply to these objects. To

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[44] Mark A. Moore in Frank G. Forrest's Valuemetricsℵ: The Science of Personal and Professional Ethics
[45] Ibid
[46] ……., 2004, The Science of Axiology. Retrieved 2007 <httm://www.ogmandino.com /Sixadvisors.htm>

value mathematics we have to know something about it. In term of the systemic value, we way we apply mathematics to the real world is to construct an abstract model of the aspect of reality we are concerned with. This can be done by introducing appropriate definitions in set theory. Mathematics can then be applied to the analysis of this model. The results are then informally translated into conclusions about the real world.

Question:

1. Explain ontology of mathematics
2. Explain ontology of science
3. Explain epistemology of mathematics
4. Explain epistemology of science
5. Explain axiology of mathematics
6. Explain axiology of science

CHAPTER TWO

THE FOUNDATIONS OF MATHEMATICS AND SCIENCE

Kompetensi Dasar Mahasiswa:

Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap persoalan-persoalan pokok dalam Pondasi Matematika dan Sain

Materi Pokok :

Persoalan-persoalan Pokok dalam Pondasi Matematika dan Sain

Strategi Perkuliahan:

Ekspositori, diskusi, presentasi, refleksi

Sumber Bahan:

Pilih yang sesuai

The search for *foundations of mathematics* is in line with the search for philosophical foundation in general. The aspects of the foundation of mathematics can be traced through the tread of philosophical history and mathematics history as well. Hersh, R. elaborates that the foundations of mathematics[47] have ancient roots; the philosophers behind Frege are Hilbert, Brouwer, Immanuel Kant. The philosopher behind Kant is Gottfried Leibniz; the philosophers behind Leibniz are Baruch Spinoza and Rene Descartes. The philosophers behind all of them are Thomas Aquinas, Augustine of Hippo, Plato, and the great grandfather of *foundationism*-Pythagoras.

The term "*foundations of mathematics*"[48] is sometimes used for certain fields of mathematics itself, namely for *mathematical logic, axiomatic set theory, proof theory* and *model theory*. The search for *foundations of mathematics* is however also the central question of the philosophy of mathematics: on what ultimate basis can mathematical statements be called *"true"*? Hersh R. describes that , the name "*foundationism*" was

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[47] Hersh, R., 1997, *"What is Mathematics, Really?"*, London: Jonathan Cape, p. 91

[48] …….., "*Foundations of mathematics*", Retrieved 2004 < *http://www.wikipedia.org/*>

invented by a prolific name-giver, *Imre Lakatos*. Gottlob Frege, Bertrand Russell, Luitjens Brouwer and David Hilbert, are all hooked on the same delusion that mathematics must have a firm foundation; however, they differ on what the foundation should be. The *currently foundation of mathematics*[49] is, characteristically as more *formalistic approach,* based on *axiomatic set theory* and formal logic; and therefore, all mathematical *theorems* today can be formulated as *theorems of set theory.*

Hersh R. (1997) writes that the *truth* of a mathematical statement, in the current view, is then nothing but *the claim that the statement can be derived from the axioms of set theory using the rules of formal logic.* However, this *formalistic*[50] *approach* does not explain several issues such as: why we should use the *axioms* we do and not some others, why we should employ the logical rules we do and not some other, why *"true"* mathematical statements appear to be true in the physical world. The above mentioned notion of *formalistic truth* could also turn out to be rather pointless; it is perfectly possible that all statements, even contradictions, can be derived from the axioms of set theory. Moreover, as a consequence of Gödel's second incompleteness theorem, we can never be sure that this is not the case.

1. Ontological Foundation of Mathematics

Relating to *ontological foundation of mathematics,* Litlang (2002) views that in *mathematical realism,* sometimes called *Platonism*, the existence of a world of mathematical objects independent of humans is *postulate*d; not our *axioms*, but the very real world of mathematical objects forms the foundation. The obvious question, then, is: *how do we access this world?* Some modern theories in the *philosophy of mathematics* deny the existence of foundations in the *original* sense. Some theories tend to focus on *mathematical practices* and aim to describe and analyze the actual working of mathematicians as a social group. Others try to create a cognitive science of mathematics, focusing on human cognition as the origin of the reliability of mathematics when applied to the *'real world'.* These theories[51] would propose to find the foundations of mathematics only in human thought, not in any *'objective'* outside construct, although it remains controversial.

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[49] Hersh, R., 1997, *"What is Mathematics, Really?"*, London: Jonathan Cape, p.91
[50] *Ibid*. p. 256
[51] *Ibid*

Litlang indicates that although mathematics might seem the clearest and most certain kind of knowledge we possess, there are problems just as serious as those in any other branch of philosophy. It is not easy to elaborate the *nature of mathematics* and in what sense do mathematics propositions have meaning?. Plato[52] believes, in *Forms or Ideas,* that there are *eternal capable* of precise definition and independent of perception. Plato includes, among such entities, numbers and the objects of geometry such as *lines, points or circles* which were apprehended not with the senses but with reason. According to Plato[53], the *mathematical objects* deal with specific instances of *ideal Forms.* Since the true propositions of mathematics[54] are true of the unchangeable relations between unchangeable objects, they are inevitably true, which means that mathematics discovers *pre-existing truths* out there rather than creates something from our *mental predispositions*; hence, mathematics dealt with *truth* and *ultimate reality.*

Litlang (2002) indicates that Aristotle disagreed with Plato. According to Aristotle, *Forms* were not entities remote from appearance but something that entered into objects of the world. That we abstract mathematical object does not mean that these abstractions represent something *remote* and *eternal*. However, mathematics is simply reasoning about *idealization*s. Aristotle[55] looks closely at the structure of mathematics, distinguishing logic, principles used to demonstrate *theorems*, definitions and hypotheses. Litlang implies that while Leibniz brought together logic and mathematics, Aristotle uses propositions of the *subject- predicate form.* Leibniz argues that the subject *contains* the *predicate*; therefore the *truths of mathematical propositions* are not based on *eternal* or *idealized entities* but based on their *denial is logically impossible.*

According to Leibniz[56], the *truth of mathematics* is not only of this world, or the world of eternal *Forms*, but also of *all possible worlds.* Unlike Plato, Leibniz sees the importance of *notation* i.e. a symbolism of calculation, and became very important in the *twentieth century mathematics* viz. a method of forming and arranging characters and signs to represent the relationships between mathematical thoughts. On the other hand, Kant[57] perceives that mathematical entities were *a-priori synthetic propositions* on which it

[52] Litlangs, 2002-2004, "*Math Theory*" Retrieved 2004 <http://www.poetrymagic.co.uk/ professional. html>
[53] *Ibid.*
[54] *Ibid.*
[55] *Ibid.*
[56] *Ibid.*
[57] Eves, H and Newsom, C.V., 1964, "*An Introduction to the Foundation & Fundamental Concepts of Mathematics*", New York: Holt, Rinehart and Winston, p.70

provides the *necessary conditions* for *objective experience*. According to Kant[58], *mathematics is the description of space and time*; mathematical concept requires only *self-consistency*, but the construction of such concepts involves *space* having a certain structure.

On the other hand, Frege, Russell and their followers[59] develop Leibniz's idea that *mathematics is something logically undeniable.* Frege[60] uses *general laws of logic* plus definitions, formulating a symbolic notation for the reasoning required. Inevitably, through the long chains of reasoning, these symbols became less intuitively obvious, the transition being mediated by *definitions.* Russell[61] sees the *definitions* as notational conveniences, mere steps in the argument. While Frege sees them as implying something worthy of careful thought, often presenting key mathematical concepts from new angles. For Russell[62], the *definitions had no objective existence*; while for Frege, it is ambiguous due to he states that the definitions are logical objects which claim an existence equal to other mathematical entities.

Eves H. and Newsom C.V. write that the *logistic thesis* is that mathematics is *a branch of logic*. All mathematical concepts are to be formulated in terms of logical concepts, and all *theorems* of mathematics are to be developed as *theorems* of logic. The distinction between *mathematics* and *logic*[63] becomes merely one of *practícal convenience;* the *actual reduction of mathematical concepts* to *logical concepts* is engaged in by Dedekind (1888) and Frege (1884-1903), and the statement of mathematical *theorems* by means of a logical symbolism as undertaken by Peano (1889-1908). The *logistic thesis* arises naturally from the effort to push down the *foundations of mathematics* to as deep a level as possible.[64] Further, Eves H. and Newsom C.V. (1964) state:

> The foundations of mathematics were established in the real number system, and were pushed back from the real number system to the natural number system, and thence into set theory. Since the theory of classes is an essential part of logic, the idea of reducing mathematics to logíc certainly suggests itself.

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[58] *Ibid.p.70*
[59] *Ibid.p.286*
[60] Litlangs, 2002-2004, "*Math Theory*" Retrieved 2004 <*http://www.poetrymagic.co.uk/ professional. html*>
[61] *Ibid.*
[62] *Ibid.*
[63] Eves, H and Newsom, C.V., 1964, "*An Introduction to the Foundation & Fundamental Concepts of Mathematics*", New York: Holt, Rinehart and Winston, p. 286
[64] *Ibid.p.286*

The logistic thesis is thus an attempted synthesization suggested why an important trend in the history of the application of the mathematical method. Meanwhile, Litlangs determines that in *geometry*, *logic* is developed in two ways. The *first*[65] is to use *one-to-one correspondences* between *geometrical entities* and *numbers. Lines, points, circle, etc.* are matched with *numbers* or *sets of numbers,* and *geometric relationships* are matched with *relationships between numbers.* The *second* is to avoid numbers altogether and define geometric entities partially but directly by their relationships to other geometric entities. Litlangs comments that such definitions are logically disconnected from perceptual statements so that the *dichotomy* between *pure* and *applied mathematics* continues. It is somewhat paralleling Plato's distinction between pure *Forms* and their *earthly copies.* Accordingly, alternative *self-consistent* geometries can be developed, therefore, and one cannot say beforehand whether actuality is or is not *Euclidean*; moreover, the shortcomings of the logistic procedures remain, *in geometry* and *in number theory.*

Furthermore, Litlangs (2002) claims that there are mathematicians perceiving mathematics as the *intuition of non-perceptual objects* and *constructions.* According to them, mathematics is introspectively *self-evident* and begins with an activity of the mind which moves on from one thing to another but keeps a memory of the first as the empty form of a common substratum of all such moves. Next, he states:

> Subsequently, such constructions have to be communicated so that they can be repeated clearly, succinctly and honestly. Intuitionist mathematics employs a special notation, and makes more restricted use of the *law of the excluded middle* viz. *that something cannot be p' and not-p' at the same time*. A *postulate*, for example, that the irrational number pi has an *infinite* number of unbroken sequences of a hundred zeros in its full expression would be conjectured as un-decidable rather than true or false.[66]

The *law of excluded middle, tertium non datur* in Latin, states that *for any proposition P, it is true that "P or not P"*. For example, if *P is "Joko is a man"* then the *inclusive* <u>*disjunction*</u> *"Joko is a man, or Joko is not a man*" is *true(* See Figure 11)*.*

P	***not P***	***P or not P***

[65] Litlangs, 2002-2004, "*Math Theory*" Retrieved 2004 <*http://www.poetrymagic.co.uk/ professio nal.html*>
[66] *Ibid.*

True	False	True
False	True	True

Figure 11 : Mathematical Table for the truth of P or not P

Litlangs (2002), further, adds that different writers perceive mathematics as simply what mathematicians do; for them, mathematics arises out of its practice, and must ultimately be a free creation of the human mind, not an exercise in logic or a discovery of preexisting fundamentals. Mathematics[67] does tell us, as Kant points out, something about the *physical world,* but it is *a physical world* sensed and understood by human beings. On the other hand[68], *relativists* remind that nature presents herself as an organic whole, with space, matter and time. Humans[69] have in the past analyzed nature, selected certain properties as the most important, forgotten that they were abstracted aspects of a whole, and regarded them thereafter as distinct entities; hence, for them, men have carried out mathematical reasoning independent of sense experience.

2. Epistemological Foundation of Mathematics

The *epistemological foundation of mathematics* elicits the status and foundation of mathematical knowledge by examining the basis of mathematical knowledge and the certainty of mathematical judgments. Nikulin D. (2004) enumerates that *ancient philosophers* perceived that mathematics and its methods could be used *to describe the natural world.* Mathematics[70] can give knowledge about things that cannot be otherwise and therefore has nothing to do with the ever-fluent physical things, about which there can only be a possibly right opinion. While Ernest P. explains that *Absolutist* philosophies of mathematics, including *Logicism*, *Formalism*, *Intuitionism* and *Platonism* perceive that

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[67] *Ibid.*
[68] Eves, H and Newsom, C.V., 1964, "*An Introduction to the Foundation & Fundamental Concepts of Mathematics*", New York: Holt, Rinehart and Winston, p.289
[69] *Ibid.p. 290*
[70] Nikulin, D., 2004, "*Platonic Mathematics*: *Matter, Imagination and Geometry-Ontology, Natural Philosophy and Mathematics in Plotinus, Proclus and Descartes*", Retrieved 2004 *<http://www. amazon.com/ exec/ obidos/AZIN/075461574/wordtradecom>*

mathematics is *a body of absolute and certain knowledge*. In contrast[71], conceptual change philosophies assert that mathematics is corrigible, fallible and a changing social product.

Lakatos[72] specifies that despite all the foundational work and development of mathematical logic, the quest for certainty in mathematics leads inevitably to *an infinite regress.* Contemporary, any mathematical system depends on a set of assumptions and there is no way of escaping them. All we can do[73] is to minimize them and to get a reduced set of *axioms* and rules of proof. This reduced set cannot be dispensed with; this only can be replaced by assumptions of at least the same strength. Further, Lakatos[74] designates that we cannot establish the certainty of mathematics without *assumptions,* which therefore is *conditional,* not absolute certainty. *Any attempt to establish the certainty of mathematical knowledge via deductive logic and axiomatic systems fails,* except in *trivial cases,* including *Intuitionism, Logicism and Formalism.*

a. ***Platonism***

Hersh R. issues that *Platonism* is the most pervasive philosophy of mathematics; today's mathematical *Platonism*s descend in a clear line from the doctrine of Ideas in Plato . Plato's philosophy of mathematics[75] came from the Pythagoreans, so mathematical "*Platonism*" ought to be "*Pythago-Platonism*." Meanwhile, Wilder R.L. contends that *Platonism*[76] is the methodological position which goes with philosophical *realism* regarding the objects mathematics deals with. However, Hersh R. argues that the standard version of *Platonism* perceives mathematical entities exist outside space and time, outside thought and matter, in an abstract realm independent of any consciousness, individual or social. Mathematical objects[77] are treated not only as if their existence is independent of cognitive operations, which is perhaps evident, but also as if the facts concerning them did not involve a relation to the mind or depend in any way on the possibilities of verification, concrete or *"in principle."*

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[71] Ernest, P, 2004. "*Social Constructivism As A Philosophy Of Mathematics:Radical Constructivism Rehabilitated?* Retrieved 2004 <*http://www.google.com/ernest*>
[72] Lakatos in Ernest, P. "*Social Constructivism As A Philosophy Of Mathematics:Radical Constructivism Rehabilitated?* Retrieved 2004 <*http://www.google.com/ernest*>
[73] *Ibid.*
[74] *Ibid.*
[75] Hersh, R., 1997, *"What is Mathematics, Really?"*, London: Jonathan Cape, p.9
[76] Wilder,R.L., 1952, *"Introduction to the Foundation of Mathematics",* New York, p.202
[77] Hersh, R., 1997, *"What is Mathematics, Really?"*, London: Jonathan Cape, pp.9

On the other hand, Nikulin D. (2004) represents that *Platonists* tend to perceive that mathematical objects are considered intermediate entities between physical things and *neotic*, merely thinkable, entities. Accordingly, *Platonists*[78] discursive reason carries out its activity in a number of consecutively performed steps, because, unlike the intellect, it is not capable of representing an object of thought in its entirety and unique complexity and thus has to comprehend the object part by part in a certain order. Other writer, Folkerts M. specifies that *Platonists* tend to believed that abstract reality is a reality; thus, they don't have the problem with truths because objects in the ideal part of mathematics have properties. Instead the *Platonists*[79] have an epistemological problem viz. one can have no knowledge of objects in the ideal part of mathematics; they can't impinge on our senses in any causal way.

According to Nikulin D., *Platonists* distinguish carefully between arithmetic and geometry within mathematics itself; a reconstruction of Plotinus' theory of number, which embraces the late Plato's division of numbers into substantial and quantitative, shows that numbers are structured and conceived in opposition to geometrical entities. In particular[80], numbers are constituted as a synthetic unity of indivisible, discrete units, whereas geometrical objects are continuous and do not consist of indivisible parts. For *Platonists*[81] certain totalities of mathematical objects are well defined, in the sense that propositions defined by quantification over them have definite truth-values. Wilder R.L.(1952) concludes that there is a direct connection between *Platonism* and the *law of excluded middle*, which gives rise to some of *Platonism*'s differences with *constructivism*; and, there is also a connection between *Platonism* and set theory. Various degrees of *Platonism*[82] can be described according to what totalities they admit and whether they treat these totalities as themselves mathematical objects. The most elementary kind of *Platonism*[83] is that which accepts the totality of natural numbers i.e. that which applies the

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[78] Nikulin, D., 2004, "*Platonic Mathematics*: *Matter, Imagination and Geometry-Ontology, Natural Philosophy and Mathematics in Plotinus, Proclus and Descartes*", Retrieved 2004 < *http://www. amazon.com/exec/ obidos/AZIN/075461574/wordtradecom*>
[79] Folkerts, M., 2004, "*Mathematics in the 17th and 18th centuries",* Encyclopaedia Britannica, Retrieved 2004 <*http://www.google.search*>
[80] Nikulin, D., 2004, "*Platonic Mathematics*: *Matter, Imagination and Geometry-Ontology, Natural Philosophy and Mathematics in Plotinus, Proclus and Descartes*", Retrieved 2004 <*http://www.amazon. com/ exec/ obidos/AZIN/075461574/wordtradecom*>
[81] Wilder, R.L., 1952, *"Introduction to the Foundation of Mathematics",* New York, p.202
[82] *Ibid.p.2002*
[83] *Ibid. p.2002*

law of excluded middle to propositions involving quantification over all natural numbers. Wilder R.L. sums up the following:

> *Platonism* says mathematical objects are real and independent of our knowledge; space-filling curves, uncountable *infinite* sets, *infinite*-dimensional manifolds-all the members of the mathematical zoo-are definite objects, with definite properties, known or unknown. These objects exist outside physical space and time; they were never created and never change. By logic's *law of the excluded middle,* a meaningful question about any of them has an answer, whether we know it or not. According to *Platonism*, mathematician is an empirical scientist, like a botanist.

Wilder R.L[84] asserts that *Platonists* tend to perceive that mathematicians can not invent mathematics, because everything is already there; he can only discover. Our mathematical knowledge[85] is objective and unchanging because it's knowledge of objects external to us, independent of us, which are indeed changeless. For Plato[86] the Ideals, including numbers, are visible or tangible in Heaven, which we had to leave in order to be born. Yet most mathematicians and philosophers of mathematics continue to believe in an independent, immaterial abstract world-a remnant of *Plato's Heaven*, attenuated, purified, bleached, with all entities but the mathematical expelled. *Platonists* explain mathematics by a separate universe of abstract objects, i*ndependent of the material universe.* But how do the abstract and material universes interact? How do flesh-and-blood mathematicians acquire the knowledge of number?

b. *Logicism*

In his *Principia Mathematica,* Irvine A.D. elaborates that *Logicism* was first advocated in the late seventeenth century by Gottfried Leibniz and, later, his idea is defended in greater detail by Gottlob Frege. *Logicism*[87] is the doctrine that Mathematics is reducible to Logic. According to Irvine A.D., *Logicism*, as the modern analytic tradition, begins with the work of Frege and Russell for both of whom mathematics was a central concern. He propounds that mathematicians such as Bernard Bolzano, Niels Abel, Louis

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[84] *Ibid.p.202*
[85] *Ibid.p.202*
[86] *Hersh, R., 1997, "What is Mathematics, Really?", London: Jonathan Cape, pp.12*
[87] Irvine, A.D., 2003, *Principia Mathematica*, Stanford Encyclopedia of Philosophy, Retrieved 2004 *<http://plato.standford.edu/cgi-bin/encyclopedia/archinfo.cgi? entry= principia mathematica>*

Cauchy and Karl Weierstrass succeed in eliminating much of the vagueness and many of the contradictions present in the mathematical theories of their day; and by the late 1800s, William Hamilton had also introduced ordered couples of reals as the first step in supplying a logical basis for the complex numbers.

Further, Irvine A.D. (2003) sets forth that Karl Weierstrass[88], Richard Dedekind and Georg Cantor had also all developed methods for founding the irrationals in terms of the rationals (see Figure 6); and using work by H.G. Grassmann and Richard Dedekind, Guiseppe Peano had then gone on to develop a theory of the rationals based on his now famous *axioms* for the natural numbers; as well as by Frege's day, it was generally recognized that a large portion of mathematics could be derived from a relatively small set of primitive notions.

For *logicists*[89], if mathematical statements are true at all, they are true necessarily; so the principles of logic are also usually thought to be *necessary* truths. Frege[90] attempts to provide mathematics with a sound logical foundation. On the other hand, Wilder R.L. persists that the effort to *reduce* mathematics to logic arose in the context of an increasing systematization and *rigor* of all pure mathematics, from which emerged the goal of setting up a comprehensive *formal system* which would represent all of known mathematics with the exception of *geometry*, insofar as it is a theory of physical space.

The goal of *logicism* [91] would then be a comprehensive *formal system* with a natural interpretation such that the primitives would be logical concepts and the *axioms* logical truths. Eves H. and Newsom C.V. (1964) maintain that Russell, in his *Principia of Mathematica* starts with primitive ideas and primitive propositions to correspond the undefined terms and *postulate*s of a *formal* abstract development. Those primitive ideas and propositions[92] are not to be subjected to ínterpretation but are restricted to intuitive concepts of logic; they are to be regarded as, or at least are to be accepted as, plausible descriptions and hypotheses concerning the real world. Eves H. and Newsom C.V. further specifies that the aim of *Principia of Mathematica* is to develop mathematical concepts and *theorems* from these *prímitive ideas* and propositions, starting with a calculus of propositions, proceeding up through the theory of classes and relations to the

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[88] *Ibid*.
[89] *Ibid.*
[90] *Ibid.*
[91] Wilder, R. L. , 1952, *"Introduction to the Foundation of Mathematics"*, New York, p.206
[92] Eves, H and Newsom, C.V., 1964, *An Introduction to the Foundation & Fundamental Concepts of Mathematics*, New York: Holt, Rinehart and Winston, p.287

establishment of the natural number system, and thence to all mathematics derivable from the natural number system. Specifically, Eves H. and Newsom C.V. ascribe the following:

> To avoid the contradictíons of set theory, *Principia of Mathematica* employs a theory of types that sets up a hierarchy of levels of elements. The primary elements constitute those of type 0; classes of elements of type 0 constitute those of type 1 ; classes of elements of type 1 constitute those of type 2; and so on.

In applying the theory of types, one follows the rule that all the elements of any class must be of the same type. Adherence to this rule precludes impredícative definitions and thus avoids the paradoxes of set theory.

As *original*ly presented in *Principia of Mathematica*, hierarehies within hierarehies appeared, leading to the so-called ramífied theory of types. In order to obtain the impredícative definitions needed to establish analysis, an axiom of reducibility had to be introduced. The nonprimitíve and arbitrary character of this axiom drew forth severe criticísm, and much of the subsequent refinement of the logistic program lies in attempts to devise some method of avoiding the disliked axiom of reducibility. [93] On the other hand, Posy C. enumerates that as a *logicist*, Cantor is not concerned with what a number; instead, he wonders of the two sets of objects which have the same number. Cantor[94] defines the notion of similarity of size i.e. equality of cardinal that two sets have the same cardinality if there exists a one to one mapping between them which exhausts them both. Cantor[95] shows *cardinality of Q = cardinality of N* by showing one to one mapping; and he found that a denumerable set is one that can be put into a one to one correspondence with the set of natural numbers.

Cantor[96] conjectures that there are only two types of *cardinal numbers*: *finite* or *infinite*; thus all *infinite* sets would be of the same size; however, he proved this conjecture false because the set of R is larger than N; in fact there are more real numbers between zero and one than there are total natural numbers. Furthermore, Posy C. indicates that the implication of Cantor's investigation of *infinity* is that there is no longer taboo to learn it and *infinity* is accepted as a notion with rich content and central to mathematics as well as that a

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[93] Posy, C. ,1992, "*Philosophy of Mathematics*". Retreived 2004 <*http://www.cs.washington.edu / homes/ gjb.doc/philmath.htm*>
[94] *Ibid.*
[95] *Ibid.*
[96] *Ibid.*

conceptual foundation for the calculus was provided that is all notions of mathematics was reduced to the ideas of natural numbers and the possibly *infinite* set. By showing *one to one mapping,* see Figure 12*,* Cantor proves that the set *N x N = {(1,1), (2,1), (1,2), (1,3), (2,2), …}* is *denumerable.*

Figure 12 : Cantor shows that N x N is denumerable

However, as Posy C.[97] claims there is also resistance of Cantor's work. Kronecker[98], for example criticizes that thought all Cantor did was nonsense because they just the artificial work of man; he wonders of mathematics has been reduced to natural numbers and sets and argues about the rigor behind natural numbers, what are natural numbers, why does the reduction stop there; and concluded that there is a general move towards creating a non-intuitive conceptual framework for natural numbers.

Still in the sphere of *logicism*, Zalta E.N. (2003) contends that Frege formulates two distinguished *formal* systems and used these systems in his attempt both to express certain basic concepts of mathematics precisely and to derive certain mathematical laws from the *laws of logic*; in his system, of 1879, he develops a second-order predicate calculus and used it both to define interesting mathematical concepts and to state and prove mathematically interesting propositions. However, in his system of 1893/1903, Frege[99] addes (as an axiom) what he thought was a distinguished logical proposition *(Basic Law V)* and tried to derive the *fundamental theorems* of various mathematical (number) systems from this proposition. According to Zalta E.N. (2003), unfortunately, not only did *Basic Law*

[97] In Posy, C. ,1992, "*Philosophy of Mathematics*". Retreived 2004 <*http://www.cs.washington.edu / homes/ gjb.doc/philmath.htm*>
[98] *Ibid.*
[99] Zalta, E.N., 2003, "*Frege's Logic, Theorem, and Foundations for Arithmetic"*, Stanford Encyclopedia of Philosophy, Retrieved 2004 <*http://plato.standford.edu/cgi-bin/encyclopedia/*>

V fail to be a logical proposition, but the resulting system proved to be *inconsistent*, for it was subject to *Russell's Paradox.*

Meanwhile, Folkerts M. (2004) designates that *Logicist* program was dealt an unexpected blow by Bertrand Russell in 1902, who points out unexpected complications with the naive concept of a set. However, as it was stated by Irvine A.D that Russell's famous of the logical or *set-theoretical paradoxes* arises within *naive set theory* by considering *the set of all sets which are not members of themselves.* Such a set appears to be a member of it self if and only if it is not a member of itself. Some sets, such as the set of teacups, are not members of themselves and other sets, such as the set of all non-teacups, are members of themselves. Russell[100] lets us call *the set of all sets* which are not members of themselves S; if S is a member of itself, then by definition it must not be a member of itself; similarly, if S is not a member of itself, then by definition it must be a member of itself. The *paradox*[101] itself stems from the idea that any coherent condition may be used to determine a set or class.

c. *Intuitionism*

The intuitionist school originated about 1908 with the Dutch mathematician L. C. J. Brouwer. The *intuitíonist thesis*[102] is that *mathematics is to be built solely by finite constructive methods on the intuitively given sequence of natural numbers.* According to this view, then, at the very base of mathematics lies a primitive intuition, allied, no doubt, to our temporal sense of before and after, which allows us to conceive a single object, then one more, then one more, and so on endlessly. In this way[103] we obtain *unending sequences,* the best known of which is the sequence of natural numbers. From this intuitive base of the sequence of natural numbers, any other mathematical object must be built *in a purely constructive manner*, *employing a finite number of steps or operations.* Important notion is expounded by Soehakso RMJT (1989) that for Brouwer, the one and only sources of mathematical knowledge is the primordial intuition of the *"two-oneness"* in which the mind

[100] Irvine, A.D., 2003, "*Principia Mathematica*", Stanford Encyclopedia of Philosophy, Retrieved 2004<*http://plato.standford.edu/cgi-bin/encyclopedia/archinfo.cgi?entry= principiamathe matica*>
[101] *Ibid*.
[102] Eves, H and Newsom, C.V., 1964, *"An Introduction to the Foundation & Fundamental Concepts of Mathematics*", New York: Holt, Rinehart and Winston, p.287-288
[103] *Ibid*.p.288

enables to behold mentally the falling apart of moments of life into two different parts, consider them as reunited, while remaining separated by time.

For Eves H. and Newsom C.V., the *intuitionists* held that an entity whose existence is to be proved must be shown to be constructible in a finite number of steps. It[104] is not sufficient to show that the assumption of the entity's nonexistence leads to a *contradiction;* this means that many existence proofs found in current mathematics are not acceptable to the *intuitionists* in which an important instance of the *intuitionists*' insistence upon constructive procedures is in the theory of sets. For the *intúitíonists*[105], a set cannot be thought of as a ready-made collection, but must be considered as a *1aw* by means of which the elements of the set can be constructed in a step-by-step fashion. This concept of set rules out the possibility of such contradictory sets as *"the set of all sets."*

Another remarkable consequence[106] of the *intuítionists'* is the insistence upon *finite constructibility,* and this is the denial of the uníversal acceptance of *the 1aw of excluded middle.* In the *Príncípia mathematica,* the *1aw of excluded middle* and the *1aw of contradiction* are equivalent. For the *intuitionists*[107], this situation no longer prevails; for the *intuitionists*, the *law of excluded middle* holds for finite sets but should not be employed when dealing with *infinite* sets. This state of affairs is blamed by Brouwer on the sociological development of logic. The *laws of logíc* emerged at a time in man's evolution when he had a good language for dealing with finite sets of *phenomena.* Brouwer then later made the mistake of applying these *laws* to the *infinite* sets of mathematics, with the result that antinomies arose.

Again, Soehakso RMJT indicates that in *intuistics mathematics,* existence is synonymous with actual constructability or the possibility in principle at least, to carry out such a construction. Hence the exigency of construction[108] holds for proofs as well as for definitions. For example let a natural number n be defined by "*n is greatest prime such that n-2 is also a prime, or n-1 if such a number does not exists*". We do not know at present whether of pairs of prime p, *p+2* is *finite* or *infinite*.

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[104] *Ibid.p.288*
[105] *Ibid.p.288*
[106] *Ibid*.p.288
[107] *Ibid.p.289*
[108] Soehakso, RMJT, 1989, "*Some Thought on Philosophy and Mathematics*", Yogyakarta: Regional Conference South East Asian Mathematical Society, p.26

The *intuitionists*[109] have succeeded in rebuilding large parts of present-day mathematics, including a theory of the continuum and a set theory, but there ís a great deal that is still wanting. So far, *intuítionist*[110] *mathematics* has turned out to be considerably less powerful than classical mathematics, and in many ways it is much more complicated to develop. This is the fault found with the *intuítionist*[111] approach-too much that is dear to most mathematicians is sacrificed. This síuation[112] may not exist forever, because there remains the possíbility of an *intuítionist* reconstruction of classical mathematics carried out in a dífferent and more successful way. And meanwhile, in spite of present objections raised against the intuitionist thesis, it is generally conceded that its methods do not lead to contradictions.[113]

d. Formalism

The *formal*ist school was founded by David Hilbert. In his *Grundlagen der Geometríe (1899)*, Hilbert[114] had sharpened the mathematical method from the material *axiomatics of Euclid* to the *formal axíomatics* of the present day. The *formalist* point of view[115] is developed by Hilbert to meet the crisis caused by the *paradoxes of set theory* and the challenge to classical mathematics caused by *intuitionistic criticism.* The *formalist thesís* is that mathematics is concerned with *formal symbolic systems.* In fact, mathematics[116] is regarded as a collection of such abstract developments, in which the terms are mere symbols and the statements are formulas involving these symbols; the ultimate base of mathematics does not lie in logic but only in a collection of prelogical marks or symbols and in a set of operations with these marks. In a *formal system,* everything is reduced to form and rule[117].

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[109] Eves, H and Newsom, C.V., 1964, *"An Introduction to the Foundation & Fundamental Concepts of Mathematics*", New York: Holt, Rinehart and Winston. P.289
[110] *Ibid.p.289*
[111] *Ibid.p.289*
[112]*Ibid.p. 289*
[113] *Ibid.p,289*
[114] Ibid. p.290
[115] *Ibid*.p.290
[116] *Ibid.p.290*
[117] Soehakso, RMJT, 1989, "*Some Thought on Philosophy and Mathematics"*, Yogyakarta: Regional Conference South East Asian Mathematical Society, p.14

Since, from the *formal*ist point of view, mathematics[118] is *devoíd* of concrete content and contains only ideal symbolic elements, the establishment of the *consistency* of the varíous branches of mathematics becomes an important and *necessary* part of the *formalíst program*. Without such an accompanying *consístent proof*[119], the whole study is essentially senseless. Eves H. and Newsom C.V. explicates the following:

> In the *formal*ist thesis we have the axíomatíc development of mathematics pushed to its extreme. The success or failure of Hilbert's program to save classical mathematics hinges upon the solution of the consistency problem. Freedom from contradiction is guaranteed only by consistency proofs, and the older consistency proofs based upon interpretations and models usually merely shift the question of consistency from one domain of mathematics to another. In other words, a consístency proof by the method of models is only relative.

Hilbert[120], therefore, conceives a new direct approach to the consistency problem; much as one may prove, by the rules of a game that certain situations cannot occur within the game. Hilbert hopes to prove, by a suítable set of rules of procedure for obtaining acceptable formulas from the basic symbols, that a contradictory formula can never occur. If one can show that no such contradictory formula is possible, then one has established the consistency of the system. Hilbert calls a direct test for *consístency* in mathematícs as proof theory. In Hilbert's view[121], it mirrors the exact movement of the mathematicians mind. For certain elementary systems, proofs of consistency were carried out, which illustrated what Hilbert would like to have done for all classical mathematics, but the problem of consistency remained refractory. It[122] is impossible for a sufficiently rich *formal*ized deductive system, such as Hilbert's system for all classical mathematics, to prove consistency of the system by methods belonging to the system.

Eves H. and Newsom C.V. ascertains that as to response that problem, this remarkable result is a consequence of an even more fundamental one, Godel proves the *incompleteness of Hilbert's system* viz. he established the existence within the system of

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[118]Eves, H and Newsom, C.V., 1964, *"An Introduction to the Foundation & Fundamental Concepts of Mathematics*", New York: Holt, Rinehart and Winston, p.289

[119] *Ibid.p.290*

[120] *Ibid*.p.290

[121] Soehakso, RMJT, 1989, "*Some Thought on Philosophy and Mathematics"*, Yogyakarta: Regional Conference South East Asian Mathematical Society, p.15

[122] Eves, H and Newsom, C.V., 1964, *"An Introduction to the Foundation & Fundamental Concepts of Mathematics*", New York: Holt, Rinehart and Winston, pp.290

"undecídable" problems, of which *consistency* of the system is one. Godel[123] saw that the *formal* systems known to be adequate for the derivation of mathematics are unsafe in the sense that their consistency cannot be demonstrated by *finitary methods formal*ized within the system, whereas any system known to be safe in this sense is inadequate. Gödel[124] showed that there was no system of *Hilbert's type* within which the integers could be defined and which was both consistent and complete. Gödel's dissertation proved the completeness of *first-order logic*; this proof became known as Gödel's Completeness Theorem. Gödel showed anything that we can represent in a *formal* system of number theory is *finitary.* Following is excerpted from Eves H. and Newsom C.V. (1964):

> According to Godel, if S be a *formal* system for number theory and if S is consistent, then there is a sentence, G, such that neither G nor the negation of G is a theorem of S. Thus, any *formal* system sufficient to express the *theorems* of number theory has to be incomplete. Gödel showed that S can prove P(n) just in case n is the Gödel-number of a theorem of S; hence there exists k, such that k is a Gödel-number of the formula P(k)=G and this statement says of itself, it is not provable.

According to Gödel, even if we define a new *formal* system *S = S + G*, we can find *G* which isn't provable in *S*; thus, *S* can prove that if *S* is consistent, then *G* is not provable. Gödel elaborated that if S can prove *Cst(S),* then *S* can prove *G*, but if *S* is consistent, it can't prove *G*, thus, it can't prove its consistency. Thus, Hilbert's Program does not work. Ultimately, one cannot prove the consistency of a mathematical theory.

e. *Structuralism*

Posy C. sets forth that according to s*tructuralist* the basic element of mathematics shouldn't be arbitrarily picked, yet nothing dictates its choice and the basic units are structures, not actually objects. This[125] leads to *structuralism* to perceive that to be a natural number is to be a place in the sequence; accordingly, if mathematics is totally abstract, why

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[123] *Ibid*.p.290

[124] Folkerts, M., 2004, "*Mathematics in the 17th and 18th centuries",* Encyclopaedia Britannica, Retrieved 2004 <http://www.google.search>

[125] Benacerraf in Stefanik, R., 1994, "*Structuralism, Category Theory and Philosophy of Mathematics*" Retrieved 2004 <*http://www.mmsysgrp.com/mathstrc.htm*>

should it have any applicability? *Structuralist*s[126] argues that mathematics is not about some particular collection of abstract objects but rather mathematics is the science of patterns of structures, and particular objects are relevant to mathematics only in so far as they instantiate some pattern or structure.

Benacerraf[127], as a *structuralist* says:

> When it comes to learning about numbers, they merely learn new names for familiar sets. They count members of a set by determining the cardinality of the set, and they establish this by demonstrating that a specific relation holds between the set and one of the numbers. To count the elements of some k-membered set b is to establish a one-to-one correspondence between the elements of b and the elements of A less than or equal to k. The relation "pointing-to-each-member-of-b-in-turn-while-saying-the-numbers-up-to-and-including-" esta blishes such a correspondence.

Benacerraf[128] concludes that there is no one account which conclusively establishes which sets are the *"real"* numbers, and he doesn't believe that there could be such an argument. According to him, any *"object"* or referent will do as long as the structural relations are maintained. Benacerraf[129] argues that Frege's belief of some "objects" for number words to name and with which numbers could be identical, stemmed from his inconsistent logic. Since all objects of the universe were on par, the question whether two names had the same referent always had a truth value; however, identity conditions make sense only in contexts where there exist individuating conditions.

Benacerraf[130] claims that if an expression of the form *"x=y"* is to have sense, it can be only in contexts where it is clear that both x and y are of some kind or category C, and that it is the conditions which individuate things as the same C which are operative and determine its truth value. According to Benacerraf[131], Frege fails to realize this fact. It is a thesis that is supported by the activity of mathematicians, and is essential to the

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[126] Posy, C. ,1992, "*Philosophy of Mathematics*". Retreived 2004 <*http://www.cs. washington.edu/ homes/ gjb.doc/philmath.htm*>
[127] Stefanik, R., 1994, *"Structuralism, Category Theory and Philosophy of Mathematics*", Retrieved 2004 <*http://www.mmsysgrp.com/mathstrc.htm*>
[128] *Ibid.*
[129] *Ibid.*
[130] Benacerraf in Stefanik, R., 1994, "*Structuralism, Category Theory and Philosophy of Mathematics"* Retrieved 2004 <*http://www.mmsysgrp.com/mathstrc.htm*>
[131] *Ibid.*

philosophical perspective underlying category theory. Benacerraf[132] concludes that numbers could not be sets at all on the grounds that there are no good reasons to say that any particular number is some particular set, for any system of objects that forms a recursive progression would be adequate.

Benacerraf[133] concludes that a system of objects exhibits the structure of the integers implies that the elements of that system have some properties which are not dependent on structure. Accordingly, it must be possible to individuate those objects independently of the role they play in the structure; however, this is precisely what cannot be done with numbers. Benacerraf[134] argues that numbers possess outside of the properties of the structure are of no consequence to the mathematician, nor should they therefore be of concern for the philosopher of mathematics. Accordingly, there is an activity which theorizes about the unique properties of individual numbers separated from the progressive structure.

According to *Structuralist*[135], *arithmetic* is the science exploring the abstract structure that all progressions have in common merely in virtue of being progressions. *Arithmetic* is not concerned with particular numbers, and there are no unique set of objects which are the numbers. Number does not have a singular reference, because the theory is elaborating an abstract structure and not the properties of individual objects. In counting, we do not correlate sets with initial segments of the sequence of numbers as extra-linguistic entities, but correlate sets with initial segments of the sequence of number words. The *recursive sequence*[136] is a sort of yardstick which we use to measure sets; questions of the identification of the referents of number words should be dismissed as misguided in just the way that a question about the referents of the parts of the ruler would be seen as misguided.

According to *Structuralist*[137], the mathematical description, model, structure, theory, or whatever, cannot serve as an explanation of a non-mathematical event without an

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[132] Stefanik, R., 1994, "*Structuralism, Category Theory and Philosophy of Mathematics*", Retrieved 2004 *<http://www.mmsysgrp.com/mathstrc.htm>*
[133] *Ibid.*
[134] *Ibid.*
[135] In Stefanik, R., 1994, "*Structuralism, Category Theory and Philosophy of Mathematics*", Retrieved 2004 *<http://www.mmsysgrp.com/mathstrc.htm>*
[136] Benacerraf in Stefanik, R., 1994, "*Structuralism, Category Theory and Philosophy of Mathematics*" Retrieved 2004 *<http://www.mmsysgrp.com/mathstrc.htm>*
[137] Shapiro in Stefanik, R., 1994, "*Structuralism, Category Theory and Philosophy of Mathematics*" Retrieved 2004 *<http://www.mmsysgrp.com/mathstrc.htm>*

account of the relationship between mathematics per se and scientific reality per se. A mathematical structure can, perhaps, be similarly construed as the form of a possible system of related objects, ignoring the features of the objects that are not relevant to the interrelations. A *mathematical structure* is completely described in terms of the interrelations; a typical beginning of a mathematical text consists of the announcement that certain mathematical objects such real numbers are to be studied. In some cases, at least, the only thing about these objects is that there are certain relations among them and/or operations on them; and one easily gets the impression that the objects themselves are not the problems. The relations and operations are what we study.

f. Constructivism

Wilder R.L.(1952) illustrates that as a complete rejection of *Platonism*, *constructivism* is not a product of the situation created by the *paradoxes* but rather a spirit which is practically present in the whole history of mathematics. The philosophical ideas taken go back at least to Aristotle's analysis of the notion of *infinity*. Kant's philosophy of mathematics can be interpreted in a *constructivist manner.* While *constructivist*[138] ideas were presented in the nineteenth century-notably by Leopold Kronecker, who was an important forerunner of *intuitionism*-in opposition to the tendency in mathematics toward set-theoretic ideas, long before the paradoxes of set theory were discovered. *Constructivist mathematics*[139] proceed as if the last arbiter of mathematical existence and mathematical truth were the possibilities of construction.

Mathematical constructions[140] are mental and derive from our perception of external objects both mental and physical. However, the passage[141] from actuality to possibility and the view of possibility as of much wider scope perhaps have their basis in intentions of the mind-first, in the abstraction from concrete qualities and existence and in the abstraction from the limitations on generating sequences. In any case, in constructive mathematics, the rules by which *infinite* sequences are generated not merely a tool in our knowledge but part of the reality that mathematics is about. *Constructivism*[142] is implied by the *postulate* that no mathematical proposition is true unless we can, in a non-miraculous way, know it true. For

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[138] Wilder, R. L. , 1952, *"Introduction to the Foundation of Mathematics",* New York, p.204
[139] *Ibid*.p.204
[140] Brouwer in Wilder, R. L. , 1952, *"Introduction to the Foundation of Mathematics",* New York, p.204
[141] Wilder, R. L. , 1952, *"Introduction to the Foundation of Mathematics",* New York, p.204
[142] Brouwer in Wilder, R. L. , 1952, *"Introduction to the Foundation of Mathematics",* New York, p.204

mathematical constructions, a proposition of all natural numbers can be true only if it is determined true by the law according to which the sequence of natural numbers is generated.

Mathematical constructions[143] is something of which the construction of the natural numbers. It is called an *idealization*. However, the construction will lose its sense if we abstract further from the fact that this is a process in time which is never completed. The *infinite*, in *constructivism*, must be potential rather than actual. Each individual natural number can be constructed, but there is no construction that contains within itself the whole series of natural numbers. A proof in mathematics[144] is said to be *constructive* if wherever it involves the mention of the existence of something and provides a method of finding or constructing that object. Wilder R.L. maintains that the *constructivist* standpoint implies that a mathematical object exists only if it can be constructed. To say that there exists a natural number x such that Fx is that sooner or later in the generation of the sequence an x will turn up such that Fx.

Constructive mathematics is based on the idea that the logical connectives and the existential quantifier are interpreted as instructions on how to construct a proof of the statement involving these logical expressions. Specifically, the interpretation proceeds as follows:

1. To prove *p or q* (\`$p \vee q$'), we must have either a proof of p or a proof of q.
2. To prove *p and q* (\`p & q'), we must have both a proof of p and a proof of q.
3. A proof of *p implies q* (\`$p \Rightarrow q$') is an algorithm that converts a proof of p into a proof of q.
4. To prove *it is not the case that p* (\`$\neg p$'), we must show that p implies a contradiction.
5. To prove *there exists something with property P* (\`$\exists xP(x)$'), we must construct an object x and prove that $P(x)$ holds.
6. A proof of *everything has property P* (\`$\forall xP(x)$') is an algorithm that, applied to any object x, proves that $P(x)$ holds. [145]

Careful analysis of the logical principles actually used in constructive proofs led Heyting to set up the *axioms* for *intuitionistic logic.* Here, the proposition $\forall n\ P(n) \vee \neg \forall n\ P(n)$ need not

[143] *Ibid.p.204*
[144] Wilder, R. L. , 1952, *"Introduction to the Foundation of Mathematics",* New York, p.204
145 Bridges, D., 1997, "*Constructive Mathematics",* Stanford Encyclopedia of Philosophy. Retrieved 2004 <http://www.douglas@hoiho.math.waikato.ac.nz>

hold even when *P*(*n*) is a decidable property of natural numbers *n*. So, in turn, the *Law of Excluded Middle (LEM)*: $p \vee \neg p$ [146]

Question:

1. Explain ontological foundation of mathematics
2. Explain epistemological foundation of mathematics
3. Explain axiological foundation of mathematics
4. Explain ontological foundation of science
5. Explain epistemological foundation of science
6. Explain axiological foundation of science

[146] *Ibid.*

CHAPTER THREE
BASIC EPISTEMOLOGICAL QUESTION

Kompetensi Dasar Mahasiswa:

Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap persoalan-persoalan pokok dalam Epistemologi

Materi Pokok :

Persoalan-persoalan Pokok Epistemologi

Strategi Perkuliahan:

Ekspositori, diskusi, presentasi, refleksi

Sumber Bahan:

Pilih yang sesuai

Kant[147] starts his thinking by asking three fundamental questions: *(1) What can I know?, (2) What should I do and (3) What may I hope for?* He tried to answer the first question in the *Critique of Pure Reason*, the second question in the *Critique of Practical Reason,* and the third question in the *Critique of Judgment*. In his critical philosophy, Kant[148] wants to find a *synthesis* of knowledge; but, unlike the medieval saint, his basis was epistemological rather than metaphysical. Kant's purpose was, in the manner of reversing the tendency and the process of modern philosophy, to criticize the validity of knowledge itself, to examine its operations, and to determine its limits. The philosophy before Kant had been emphasizing on the knowledge of the objects of the external world,

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[147] *Ibid*.p.294
[148] *Ibid. p.294*

but Kant lays the stress on cognition and the way objects are determined by our understanding.

Kant[149] states that if we want to understand the nature of the universe, we must look at man's mind. Due to the human mind is still the subject to limitations, it cannot be an absolute key of reality. Although the human mind cannot supply the content of experience, it can give us the forms in which we perceive it. Kant[150] calls his philosophy *transcendental* viz. that he is concerned not so much about *phenomena* as with our *a priori* knowledge of them. However he prefers to find out in what way our minds deal with the objects of the external world. Above all, Kant[151] wants to set forth the *a priori principles* which are fundamental in any epistemological investigation. Therefore, Kant's theory of knowledge is based on this *a priori principles* and on the *synthatical* judgment.

Kant[152] went into every aspect of all the relevant problems attempted by previous philosophers; and thus, Kant's works are found as repetitions of all earlier attempts to solve these problems. Kant's fundamental question concerning epistemology is: How are *synthetical* judgments *a priori* possible? According to Kant[153], the solution of the above problem is comprehended at the same *time* toward the possibility of the use of *pure* reason in the foundation and construction of all sciences, which contain theoretical knowledge *a priori* of objects; and upon the solution of this problem, depends on the existence or downfall of the science of metaphysics. Accordingly, a system of absolute, certain knowledge can be erected only on a foundation of judgments that are *synthetical* and acquired independently of all experiences.

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[149] *Ibid.p.295*
[150] *Ibid.p.295*
[151] *Ibid.p.295*
[152] Steiner, R., 2004, "*Truth and Knowledge: Kant's Basic Epistemological Question",* The Rudolf Steiner Archive, Retreived 2004<*http://www.elibrarian@elib.com*>
[153] *Ibid.*

By the use of simple illustrations, Kant[154] shows that *synthetic* judgments *a priori* are fundamental in mathematics, physical science, and metaphysics. For example[155], in mathematics we say that three plus four is seven. How do we know this? It's not by experience but by *a priori* knowledge. Moreover, we express a necessity in this judgment; past knowledge has shown that three plus four is seven, but we assert that the same case must hold true for the future. Kant[156] calls a judgment as *synthetical* where the concept of the predicate brings to the concept of the subject of something which lies completely outside the subject. Although it stands in connection with the subject, however, in analytical judgment, the predicate merely expresses something which is already contained in the subject.

Kant[157] claims that knowledge in the form of judgment can only be attained when the connection between predicate and subject is *synthetical* in this sense; and it demands that these judgments must be acquired *a priori*, that is independent of all experiences. Two *presuppositions*[158] are thus found in Kant's formulation of the questions; first, is that we need other means of gaining knowledge besides experience, and second, is that all knowledge gained through experience is only approximately valid. It does not occur to Kant[159] that the above principles need proof that is open to doubt and they are prejudices which he simply takes over from dogmatic philosophy and then uses them as the basis of his critical investigations. He made the same assumptions and merely inquired under what conditions that they are valid or not valid.

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[154] Mayer, F., 1951, "*A History of Modern Philosophy*", California: American Book Company, p.296
[155] *Ibid. p. 296*
[156] Steiner, R., 1004, "*Truth and Knowledge: Kant's Basic Epistemological Question*", The Rudolf Steiner Archive, Retreived 2004<*elibrarian@elib.com*>
[157] Kant, I., 1787, "*The Critique of Pure Reason: Preface To The Second Edition*", Translated By J. M. D. Meiklejohn, Retrieved 2003<http://www.encarta.msn. com/>
[158] Steiner, R., 1004, "*Truth and Knowledge: Kant's Basic Epistemological Question*", The Rudolf Steiner Archive, Retreived 2004<*elibrarian@elib.com*>
[159] *Ibid.*

Cohen and Stadler in Steiner R. attempted to prove that Kant has established *a priori* nature of mathematical and *pure*ly scientific principles. However[160], Kant in the *Critique of Pure Reason* attempted to show that mathematics and *pure* natural science are *a priori* sciences, in which the form of all experiences must be inherent in the subject itself and the only thing left is the material of sensations. Kant[161] builds up the material of sensations into a system of experiences in the form of which is inherent in the subject. Kant[162] claims that the formal truths of *a priori* theories have meaning and significance only as principles which regulate the material of sensation and they make experience possible, but do not go further than experience. Kant[163] concludes that these formal truths are the *synthetical judgment a priori*, and they must, as condition necessary for experience, extend as far as the experience itself.

The capital feature[164] in *Kant's Criticism of the* Judgment is that in his giving a representation and a name to the idea. Such a representation[165], as an *intuitive understanding* or an *inner adaptation*, suggests a universal which is at the same *time* apprehended as essentially a concrete unity. The principle[166], by which the *reflective faculty of judgment* regulates and arranges the products of animated nature, is described as the *End* or *final* cause of the notion in action in which the universal at once determinates in itself. According to Kant[167], reason can know *phenomena* only, there would still have been an option for animated nature between two equally subjective modes of thought. Even, according to Kant's own exposition, there would have been an obligation to admit, in the

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[160] In Steiner, R., 2004, "*Truth and Knowledge: Kant's Basic Epistemological Question*", The Rudolf Steiner Archive, Retreived 2004<*elibrarian@elib.com*>
[161] Ibid.
[162] *Ibid.*
[163] *Ibid.*
[164] Hegel, G.W.F, 1873, "*The Critical Philosophy: from The Shorter Logic*", translated by William Wallace, Clarendon Press. Retrieved 2004 <http://www.marxists.org/reference/subject/philosophy/index.htm>
[165] *Ibid.*
[166] *Ibid.*
[167] *Ibid.*

case of natural productions, a knowledge is not confined to the *categories* of quality, cause and effect, composition, constituents, and so on.

The principle of inward adaptation or design[168] had been kept to and carried out in scientific application and would have led to a different and higher method of observing nature. Thus, Kant's epistemology did not seek to obtain knowledge of the object itself, but sought to clarify how objective truthfulness can be obtained. He names it the *transcendental* method. For Kant[169], cognition is judgment. Judgment is made in terms of a proposition, and in a proposition there are subject and predicate. Knowledge increases through a judgment, in which a new concept that is not contained in the subject appears in the predicate. Kant[170] calls such a judgment "*synthetic judgment.*" In contrast, a judgment in which the concept of the predicate already contained in the concept of the subject is called "*analytical judgment*."; in the end, new knowledge can be obtained only through *synthetic judgments.*

Although new knowledge[171] can be obtained through *synthetic* judgment, it cannot become correct knowledge if it does not have *universal validity*. In order knowledge to have *universal validity*, it should not be merely *empirical* knowledge, but should have some *a priori* element independent of experience. In order a *synthetic* judgment to have *universal validity*, it must be an *a priori* cognition, namely, *a priori synthetic* judgment. So, Kant[172] had to cope with the question: How are *a priori synthetic judgments possible?;* and Kant solved this question in three fields: mathematics, physics, and metaphysics; and the three main divisions of the first part of the Critique deal respectively with these.

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[168] *Ibid*.
[169] Kant, I., 1787, "*The Critique of Pure Reason: Preface To The Second Edition*", Translated By J. M. D. Meiklejohn, Retrieved 2003<*http://www.encarta.msn. com/*>
[170] *Ibid.*
[171] *Ibid*.
[172] *Ibid.*

As for Kant[173], the central problem of his philosophy is the *synthetic a priori* knowledge or judgment; Kant beliefs that all knowledge are *reducible* to the forms of judgment. Knowledge[174] is obtained by judgments. There are two judgments. First, *synthetic* judgments i.e. judgments which expand our knowledge of nature or *analytic* judgments i.e. mere explications or explanations of what we already know. Second, *a priori* judgments i.e. knowledge which are universally and necessarily valid or *a posteriori* judgments i.e. judgments which are merely subjective and do not possess the *apodeicticity*. Kant[175] advocates that de facto there are *synthetic a priori* judgments in arithmetic, geometry, physics and metaphysics. These sciences are not only possible, but also actual as our universal and necessary knowledge.

According to Kant[176], in its *synthetic a priori* form all the laws and knowledge of those sciences are explicitly stated; however, there are differences between the *pure* mathematics, *pure* natural sciences and metaphysics. Seeing the former, we can ask only how they are possible at all. For we have evidence[177] while in the latter, we must ask how *synthetic a priori* knowledge is possible at all. How is *pure* mathematics possible? Kant claims it is possible because it is *pure a priori intuition.* How is *pure* physics possible? He claims it is possible because there are *categories*. How is metaphysics as natural faculty possible? He claims it is possible because there are *concepts of reason.* How is metaphysics as a science possible? He claims it is possible as *Practical Sciences.*[178]

Question:

Explain basic epistemological questions?

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173 *Ibid*.
174 *Ibid.*
175 *Ibid.*
176 *Ibid*.
177 *Ibid.*
178 Ibid.

CHAPTER FOUR
THE SYSTEM OF TRANSCENDENTAL ANALYTIC

Kompetensi Dasar Mahasiswa:
Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap persoalan-persoalan pokok dalam Sistem Transendental

Materi Pokok :
Persoalan-persoalan Pokok Sistem Transendental

Strategi Perkuliahan:
Ekspositori, diskusi, presentasi, refleksi

Sumber Bahan:
Pilih yang sesuai

In the *Critique of Pure Reason*, Kant, claims that *pure* understanding is the source of all principles, rules in respect of that which happens, and principles according to everything that can be presented to us as an object must conform to rules. Accordingly, Mathematics is made up of *pure a priori principles* that we may not ascribe to the *pure* understanding which is the faculty of concepts. Kant[179] claims that not every kind of knowledge *a priori* should be called *transcendental* ; only that by which we know that certain representations can be employed or are possible *a priori*; and *space* is the knowledge that the representations are not *empirical*. Kant[180] notes that the distinction

[179] Kant, I., 1787, "*The Critique of Pure Reason: Preface To The Second Edition*", Translated By J. M. D. Meiklejohn, Retrieved 2003<http://*www.encarta.msn*. com/>
[180] *Ibid.*

between *transcendental* and *empirical* belongs only to the critique of knowledge, not to the relation of that knowledge to its objects.

1. Discovery of all Pure Concepts of the Understanding

Kant[181] perceives truth as agreement of knowledge with its object and the general criterion must be valid in each instance regardless of how objects vary. Since truth concerns the content, a sufficient and general criterion cannot be given. Wallis[182] explores the progressive stages of Kant's analysis of the faculties of the mind which reveals the *transcendental* structuring of experience. First, in the analysis of *sensibility*, Kant argues for the necessarily spatiotemporal character of sensation; and then Kant analyzes the understanding, the faculty that applies concepts to sensory experience. Kant[183] concludes that the "*categories*" provide a necessary, foundational template for our concepts to map onto our experience. In addition to providing these *transcendental* concepts, the understanding is also the source of ordinary *empirical* concepts that make judgments about objects possible. The understanding provides concepts as the rules for identifying the properties in our representations.

According to Kant[184], all combination of an act of the understanding is called *synthesis* because we cannot apply a concept until we have formed it; and therefore, '*I think*' must accompany all my representations. Intuition[185] in which representation can be given prior to all thought, has a necessary relation to '*I think*' and is an act of *spontaneity*

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[181] *Ibid*.
[182] Wallis, S.F, 2004, "*Immanuel Kant (1724-1804)"*, New York: Media & Communication, The European Graduate School. Retreived 2004 <http://www.uni.scrf.ucam.org/ccps/Kantian.pdf>
[183] *Ibid.*
[184] Kant, I., 1787, "*The Critique of Pure Reason: Preface To The Second Edition*", Translated By J. M. D. Meiklejohn, Retrieved 2003<*http://www.encarta.msn. com/*>
[185] *Ibid.*

that cannot belong to *sensibility*. The identity[186] of the apperception of a *manifold* which is given in intuition contains a *synthesis* of representations, and is possible only through the consciousness of this *synthesis*. The analytic unity of apperception[187] is possible only under the presupposition of a certain *synthetic unity* of the *manifold of intuition.* Kant[188] claims that through the *'I'* as simple representation, no *manifold* is given; for a *manifold* is given in intuition which is distinct from the '*I'* and only through combination in one consciousness it can be thought.

Kant[189] insists that the supreme principle of the possibility of all intuition in relation to *sensibility* is that all the *manifold* of intuition should be subject to the formal conditions of *time* and *space*; while, the supreme principle of the same possibility in its relation to the understanding is that the *manifold* of intuition should be subject to the conditions of the original *synthetic* unity of apperception. Ross, K.L. (2001) exposes that Kant proposes that *space* and *time* do not really exist outside of us but are *"forms of intuition,"* i.e. conditions of perception, imposed by our own minds. While Gottfried, P (1987) notes from Kant that although the forms of *time* and *space* are *subjective conditions of sensation* and depend on their appearance of perceptual activity, they are nonetheless characterized as being *a priori*. They are antecedent to the specific sensations for which they provide a conceptual frame.

Kant[190] states that *time* existed is not for itself or as an objective quality in things; to conceive of *time* as something objective would require its presence in things which were not objects of perception. However, since *time* and *space* are only knowable as the *a priori* forms of intuition, any other assumption about them, apart from this context, could not be

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[186] *Ibid.*
[187] *Ibid.*
[188] *Ibid.*
[189] *Ibid.*
[190] Kant, I., 1787, "*The Critique of Pure Reason: Preface To The Second Edition*", Translated By J. M. D. Meiklejohn, Retrieved 2003<*http://www.encarta.msn. com/*>

substantiated. According to Kant[191], *time* is also the form of our inner sense, of our intuition of ourselves and of our own inner situation; belonging neither to any pattern nor place, it determines the relationship of perceptions within our inner situation; because this inner intuition as such assumed no shaper, it had to be imagined by positing *succession* through a line extending *ad-infinitum* in which sensory impressions form a *uni-dimensional* sequence and by generalizing from the attributes of this line to those of *time* itself.

2. The Deduction of the Pure Concepts of Understanding

Kant, 1787, strives to demonstrate that *space* and *time* are neither experience nor concepts, but they are *pure* intuition. He calls it as *metaphysical demonstrations* of *space* and *time*; and concludes that: firstly, *space* is not an *empirical* concept obtained by abstraction due to any *empirical* concept obtained from the external senses such as even "*next to each other"* presupposes the notion of *space*; and this means that two things are located at two different *space*s. *Time*[192] is not obtained by abstraction or association from our *empirical* experience, but is prior to the notion of simultaneous or *successive*. *Space* and *time* are anticipations of perception and are not the products of our abstraction.

Secondly[193], the idea of *space* is necessary due to the fact that we are not able to think of *space* without everything in it, however we are not able to disregard *space* itself. We[194] can think of *time* without any *phenomenon*, but it is not possible to think of any *phenomenon* without *time*; *space* and *time* are *a priori* as the conditions for the possibility of *phenomena*. Thirdly[195], the idea of *space* is not a universal concept; it is an individual idea or an intuition. There is only one *time* and various special *time*s are parts of the whole *time*

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[191] *Ibid.*
[192] *Ibid*.
[193] *Ibid*.
[194] *Ibid.*
[195] *Ibid.*

and the whole is prior to its parts. Fourthly, *space* is infinite and contains in itself infinitely many partial *space*s.

Next, Kant, 1787, develops *Transcendental Demonstrations* to indicate that the possibility of *synthetic a priori* knowledge is proven only on the basis of *Space* and *Time*, as follows: first, if *space* is a mere concept and not an intuition, a proposition which expands our knowledge about the characters of *space* beyond the concept cannot be analyzed from that concept. Therefore, the possibility of *synthesis* and expansion of Geometric knowledge is thus based on *space*'s being intuited or on the fact that such a proposition may be known true only in intuition. And thus the truth of a Geometric proposition can be demonstrated only in intuition.

Second[196], the *apodeicticity* of Geometric knowledge is explained from the apriority of intuition of *space* and the *apodeicticity* of Arithmetics knowledge is explained from the apriority of intuition of *time*. If *space* and *time* are to be *empirical*, they do not have necessity; however, both Geometric and Arithmetic propositions are universally valid and necessary true. Third[197], mathematical knowledge has the objective reality that based on *space* and *time* in which our experiences are possible. Forth, in regard to *time*, change and motion are only possible on the basis of *time*.

3. The Schematism of the Pure Concepts of Understanding

Kant, 1787, claims that as a one-dimensional object, *time* is essentially *successive* that is one moment follows another; and in order to think *time* as a *succession*, we must generate the *time-series* i.e. we must think one moment as following another. Kant[198] suggests that at each point of the series up to that point; therefore, we always think *time* as

[196] *Ibid*.
[197] Ibid.
[198] *Ibid*.

a magnitude. Accordingly, since the *categories* of quantity are those of unity, plurality and totality, we can say that they apply to appearances in that all appearances must be thought as existing within a specific *time-span* which can be thought as momentary, that is, as a series of *time* spans or as the completion of a series of *time* spans.

On the other hand, Kant[199] insists that we can think of a given *time* as either empty or full; in order to represent objects in *time* we must resort to sensation, so that in thinking a *time* we must always ask whether that *time* is filled up. Thus the *schema* of quality is the filling of *time*; it would be natural to assume that the question whether-a *time* is full admits of a simple answer of yes or no. However, Kant[200] claims that reality and negation must be conceived as two extremes or limits, between which exist infinitely many degrees; he calls these degrees as "*intensive magnitudes*"

Meanwhile, Kant, 1787, insists that *time* is supposed to relate objects, not to one another, but to the understanding, that is, we can think an object in one of three ways: (1) as occupying some *time* or other without specifying what part of *time*, that is, the *schema of possibility* in which we can think of an object as possible in so far as we can think of it as occupying some *time* or other, whether or not it actually occupies it; (2) as existing in some definite *time* that is the *schema* of *actuality* in which we think of an object as actual when we claim that it exists in some specific part of *time*; and (3) as existing at all *time*s that is the *schema* of necessity in which an object is thought as being necessary if it is something which we must represent as occupying all *time*s. In other words, we could not think of a *time* which does not contain that object.

[199] *Ibid*.
[200] *Ibid*.

Kant[201] sums up that *time* is to be seen as the formal *a priori* condition for all appearance; whereas *space* remains the *pure form* of all outward intuition, *time* supplied the subject with an inward orientation essential for perceptual relations. Kant[202] argues that the structure for the *a posteriori* representations we receive from sensation must itself be *a priori*. This leads him to the science of *a priori sensibility*, which suggests that our capacity to receive representations of objects includes a capacity to receive representations of the *a priori* form of objects. Accordingly, since *space* is one of two such *a priori* forms, *a priori sensibility* includes a capacity to receive *pure* representations of *space*. Kant[203] denies that *time* and *space* as an absolute reality, and maintains that outside of its cognitive function *time* is nothing. Accordingly, the objective validity of *time* and *space* is limited to the regularity of their relationship to sensation; yet within this limited framework, their activity was constant and predictable.

Kant[204] states that *space* and *time* do not exist by *themselves,* that is, they are not real things existing outside of our mind. They are not qualities, nor relations belonging to the *things in themselves*. They are the forms of our *empirical* intuition and are rooted in the subjective structure of our mind. Further, he claims that we sense *space* and *time* with two forms of *empirical intuition* and *they themselves intuition* at the same *time*. These intuitions are *pure*, since they are capable of becoming objects of our inquiry quite apart and independent from our *empirical* intuition. Kant[205] also claims that *space* and *time* are also *a priori*, because these intuitions as the forms of *empirical* intuitions precedes from all

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[201] Gottfried, P., 1987, "*Form of Intuition: Kantian Time And Space Reconsidered*", The World & I: Issue Date: AUGUST 1987 Volume:02 Page: 689. Retrieved 2004 <*http://www.worldandi.com/public/1987/ august/copyright.asp*>

[202] Shabel, L., 2003, "*Reflections on Kant's concept (and intuition) of space*", Studies In History and Phi losophy of Science Part A Volume 34, Issue 1 Retreived 2003, <*http://www. sciencedirect. com/science?*>

[203] Gottfried, P., 1987, "*Form of Intuition: Kantian Time And Space Reconsidered*", The World & I: Issue Date: AUGUST 1987 Volume:02 Page: 689. Retrieved 2004 <*http://www.worldandi. com/public/1987/ august/copyright.asp*>

[204] -------, "Immanuel Kant (1724-1804): "*Kant's Criticism against the Continental Rationalism and the British Empiricism*". Retrieved 2004 <*http://www.Google. com/Kant*>

[205] *Ibid.*

empirical intuitions, as long as they are the subjective conditions in which something can be an object of our *empirical* intuition.

Space and *time*[206], therefore, are not containers in which all the real things are encompassed nor the dimension or order which belongs to the *things in themselves*; they are the forms of our intuition. Kant[207] claims that our ideas are in regard to their origin either *pure* or *empirical*; they are intuitions or concepts. While Evans, J.D.G, (1999), notes from Kant that the notion of object *structurally* presupposes the subject, so the *transcendental* and necessary unity of apperception is the end product of a process of connection and *synthesis* of *phenomena* which depends on the application of the representation of an object in intuition to experience. Our minds are not comfortable with simply observing the sensuous world and its connections through universal laws; it requires some knowledge of *things in themselves* to be content (Kolak, in Meibos, A.). We know that *pure* science exists because there are universal laws, such as "*substance is permanent*" and "every event is determined by a cause according to constant laws"

These laws[208] must not be *a posteriori*, because experience can only teach us what exists and how it exists, but not that it must exist. Neither are they *a priori*, for we must make our deductions from observations. However, the conformity of experience to constant laws must be an *a priori* understanding. Through our awareness[209], we have perceptions; then, our *sensibility*, by using the concepts of *pure* understanding, structures these perceptions into experiences which we use to form science. This process is called the *schematism of pure understanding* where *schemata* are notions of objects categorized and structured in *time*. The *categories* can only subsume *schemata* and not awareness.

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[206] *Ibid*.
[207] *Ibid*.
[208] Meibos, A., 1998, "*Intro to Philosophy: Kant and a priori Synthetic Judgment*"*s,* Prof. Arts Notes for PHIL 251. Retrieved 2004 <*http://www.icecavern.net/~qirien/punkus/index.html*>
[209] *Ibid*.

Kant[210] claims that there is only one way in which a mediating element can be discovered, that is, by examining the single element which is present in all appearances, but at the same *time,* it is capable of being conceptualized as "*time*". According to him, we must, therefore, discover various ways of thinking of *time*, and if we can discover the ways in which this must be done, we can say that they both conform to the conditions of thought and are present in all appearances. Kant[211] calls these conceptualizations of *time* "*schemata*". He then finds four fundamental modes of thinking *time*, one corresponding to each of the basic divisions of *categories* that are *time*-series, *time*-content, *time*-order, and the scope of *time*. Kant[212] convicts that *schemata* for the *categories* of relation are treated separately because the relational *categories* treat them in respect to one another and that *time* considered of *it-self* is *successive* but not *simultaneous*, and *space* is *simultaneous* but not *successive*.

Kant[213], therefore thinks objects in a *time*-order: as enduring through a number of *time*s i.e. of the *permanence* of substance; as abiding while all else change; as in one state of affairs which succeeds another i.e. we think the states of substances as occupying a *succession* of *time*s in accordance with a rule; and as co-existing i.e. the *schema* of reciprocity or mutual simultaneous interaction. Next, Kant maintains that in all subsumptions under a concept, the representation must be homogeneous with the concept; however *pure concepts of understanding* can never be met with any intuition. Hence, Kant argues that the *transcendental schema* in which it mediates principle between category and appearances must be *pure* and yet sensible.

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[210] Kant, I., 1787, "*The Critique of Pure Reason: Preface To The Second Edition*", Translated By J. M. D. Meiklejohn, Retrieved 2003<*http://www.encarta.msn. com/*>
[211] *Ibid.*
[212] *Ibid*
[213] *Ibid.*

According to Kant[214], the application of the category to appearances becomes possible by means of the *transcendental* determination of *time*, that is, the *schema* of the concepts of understanding and mediates the subsumption of appearances under the category. Accordingly, the *schema* is always a product of imagination; it makes images possible as the products of the *empirical* faculty of reproductive imagination. Kant[215] concludes that there is a *schema* for each category in which the *magnitude* is the *generation of time* itself in the *successive* apprehension of an object. Kant[216] defines *quality* as the filling of *time* and *reality* as the sensation in general pointing to being in *time*; while *negation* is not-being in time and relation is the connecting of perceptions at all times according to a rule of *time* determination.

Further, *substance*[217] is permanence of the real in *time*; *cause* is the real which something else always follows; *community* is the *coexistence* according to a universal rule of the determinations of one substance with those of another. While *modality*[218] is the *time* itself as the correlation of the determination whether and how an object belongs to *time*; *possibility* is the agreement of the *synthesis* of different representations with the conditions of *time* in general; *actuality* is the existence in some determinate *time* and the *necessity* is the existence of an object at all *time*s.

4. System of all Principles of Pure Understanding

Propositions, according to Kant, 1787, can also be divided into two other types: *empirical* and *a priori*; *empirical* propositions depend entirely on sense perception, but *a priori* propositions have a fundamental validity and are not based on such perception.

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[214] *Ibid*.
[215] *Ibid*.
[216] *Ibid*.
[217] *Ibid*.
[218] *Ibid*.

Kant's claims[219] that it is possible to make *synthetic a priori judgments* and regards that the *objects of the material world* is fundamentally unknowable; therefore, from the point of view of reason, they serve merely as the raw material from which sensations are formed. Kant[220] maintains that the *category* has no other application in knowledge than to objects of experience. To think an object and to know an object are different things. Accordingly, *knowledge* involves two factors: the *concept* and the *intuition.* For the only intuition possible to us is sensible, the thought of an object can become knowledge only in so far as the concept is related to objects of the senses. This determines the limits of the *pure concepts of understanding.*

Kant[221] insists that since there lies in us a certain form of *a priori* sensible intuition, the *understanding*, as spontaneity, is able to determine inner sense through the *manifold* of given representations in accordance with the *synthetic* unity of apperception. In this way the *categories* obtain objective validity. Further Kant[222] insists that *figurative synthesis* is the synthesis of the manifold which is possible and necessary a priori. It opposes to combination through the understanding which is thought in the mere category in respect to intuition in general. It may be called the *transcendental synthesis of imagination* that is the faculty of representing in intuition of an object which is not present; and of course it belongs to *sensibility*.

For the principle that all intuition are extensive, as it was elaborated in the *Critique of Pure Reason*, Kant, 1787, proves that all *appearances* are *extensive magnitudes* and *consciousness* of the synthetic unity of the manifold is the concept of *magnitude*. A *magnitude* is extensive when the representation of the parts makes possible and therefore necessarily precedes the representation of the whole. In *appearances*, the real i.e. an

[219] *Ibid*.
[220] *Ibid.*
[221] *Ibid*.
[222] *Ibid.*

object of sensation, has intensive magnitude or a degree. Kant[223] proves that *perception* is empirical consciousness and *appearances* are not pure intuition like *time* and *space*. They[224] contain the real of sensation as subjective representation. Therefore, from *empirical consciousness* to *pure consciousness* a graduated transition is possible. There is also possible a *synthesis* in the process of generating the magnitude of a sensation as well as that the sensation is not itself an objective representation. Since neither the intuition of *space* nor *time* has met with it, its magnitude in not extensive but intensive.

Kant[225] proves that *experience* is possible only through the representation of a necessary connection of perceptions. For *experience* is an empirical knowledge, it is a synthesis of perceptions; it is not contained in perception but containing itself in one consciousness of the synthetic unity of the manifold of perceptions. And since *time*[226] itself cannot be perceived, the determination of the existence of objects in *time* can take place only through their relation in *time* in general. Since this determination always carry a necessity with time, experience is only possible through a representation of necessary connection of perceptions. Kant[227] ascertains that the three *modes of time* are *duration, succession,* and *coexistence* and the general principles of the *three analogies* rest on the necessary unity of apperception at every instant of *time*. These general principles are not concerned with appearances but only with existence and relation in respect to *existence*. *Existence*, therefore, can never be known as *a priori* and can not be constructed like mathematical principles so that these principles will be only regulative. These *analogies* are valid for empirical employment of understanding but not for transcendental one. In the principle, we use the *category*; but in its application to appearances, we use the *schema.*

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[223] *Ibid.*
[224] *Ibid*
[225] *Ibid.*
[226] *Ibid.*
[227] *Ibid.*

5. Phenomena and Noumena

According to Kant[228], *transcendental illusion* is the result of applying the understanding and sensibility beyond their limits. Although the objective rules may be the same in each case, the subjective idea of *causal connection* can lead to different deductions. Kant[229] indicates that reason which connects us directly to *things in themselves* is a question that he cannot answer. *Transcendental Deduction* aimed at showing that *particular concepts*, like causality or substance, are *necessary conditions* for the possibility of experience. Since *objects*[230] can only be experienced *spatio-temporally,* the only application of concepts that yields knowledge is to the empirical spatiotemporal world. Beyond that realm, there can be no sensations of objects for the understanding to judge rightly or wrongly.

Kant[231] states that *thoughts* without *content* are *empty*; *intuitions* without *concepts* are *blind*. To have meaningful awareness some datum is required. Accordingly, we possess two sources of input that can serve as such a datum physical sensation and the sense of moral duty. Kant[232] admits that *transcendental synthesis* of imagination is an action of the understanding on sensibility, first application, and the ground of all other applications of the understanding. Kant[233] finds that there was a *paradox* of how *inner sense* can represent to consciousness ourselves as we appear to ourselves. This *paradox* is coming from the fact that the understanding is able to determine sensibility inwardly. The understanding

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[228] Meibos, A., 1998,*" Intro to Philosophy: Kant and a priori Synthetic Judgments,"* Prof. Arts Notes for PHIL 251 Retrieved 2004 <http://www.icecavern.net/~qirien/punkus/index.html>
[229] Evan, J.D.G., 1999, *"Kant's Analysis of the Paralogism of Rational Psychology in Critiqueof Pure Reason Edition B, Kantian Review vol. 3 (1999), 99-105*. Retrieved 2004 < http://www.qub.ac.uk */phil/courses/Kant*>
[230] Wallis, S.F, 2004, *Immanuel Kant (1724-1804),* New York: Media & Communication, The European Graduate School. Retreived 2004 <*http://www.uni.scrf.ucam.org/ccps/Kantian.pdf*>
[231] *Ibid*.
[232] Kant, I., 1787, "The Critique of Pure Reason: Preface To The Second Edition", Translated By J. M. D. Meiklejohn, Retrieved 2003<*http://www.encarta.msn. com/*>
[233] *Ibid.*

performs this act upon the passive subject whose faculty it is. While the understanding does not find in inner sense a combination of the manifold, we intuit inner sense of ourselves only as we are inwardly affected by ourselves.

Kant[234] claims that in the synthetic original unity of apperception, we are conscious only that we are. This is a thought, not an intuition. The consciousness of self is very far from being a knowledge of self; it also needs an intuition of the manifold in the self. According to Kant[235], the *transcendental deduction* of the universally possible employment in experience of the pure concepts of the understanding needs to be clarified that the possibility of knowing a priori, by means of the categories of whatever objects, present *themselves* to our senses in respect of the laws of their combination. On the other hand, Kant[236] points out that the relations in which *a priori* is recognizable in *space* and *time* are valid to all the possible objects of experience. However, they are valid only to the *phenomena* and not to the *things in themselves*. Therefore, *space* and *time* have the *empirical reality* and the *transcendental ideality* at the same time.

Kant[237] insists that *any thing* as long as it is an *external phenomenon* necessarily appears in *spatial relationship*; while any *phenomenon* is necessarily appears in *temporal relationship*. It[238] calls that *space* and *time* are *objective* to everything given in experience; therefore, *space* and *time* are *empirically real*. They do not have *absolute reality* because they do not apply to *things in themselves*, whether as substances or as attributes. Due to *space* and *time* have no reality, but they are ideal, this, then, is called the *Transcendental Ideality* of *Space* and *Time*. Kant[239] contends that we are never able to recognize *things in*

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[234] *Ibid.*
[235] *Ibid.*
[236] *Ibid.*
237……"Immanuel Kant (1724-1804) "*Kant's Criticism against the Continental Rationalism and the British Empiricism*" Retrieved 2004 <*http://www.Google.com/Kant?*>
[238] *Ibid.*
[239] *Ibid.*

themselves. *Any quality* which belongs to the *thing- in- itself* can never be known to us through senses. At the same time, anything which given in *time* is not the *thing- in- itself*. What we intuitively recognize ourselves by reflection, is how we appear as a *phenomenon*, and not how we really are.

Kant[240] claims that *synthesis of apprehension* is the combination of the manifold in an empirical intuition. *Synthesis of apprehension* of the manifold of appearance must conform to *time* and *space*. *Time* and *space*[241] are themselves intuitions which contain a manifold of their own. They are not presented in a priori and they are not just the forms of sensible intuitions. *Unity* of synthesis of the manifold[242] i.e. a combination to which everything conformly represented in *space* and *time*, is given *a priori* as the condition of the synthesis of all apprehension, without or within us, not in, but with these intuitions. Kant then concludes that all *synthesis* was in subject to the *categories* in which it prescribes *laws of a priori* to appearances. They do not exist in the appearances but only relative to the subject.

Kant[243] claims that *pure understanding* is not in a position to prescribe through *categories* any *a priori laws* other than those which are involved in a nature in general that is in conformity to *space* and *time*. *Empirical laws* cannot be derived from *categories* but are subject to them. In term of the outcome of this *deduction* of the concepts of understanding, according to Kant, we cannot think of an object safe through the *categories* and cannot know an object so thought safe through intuitions corresponding to these concepts. For all our *intuitions* are empirical, there can be no *a priori* knowledge except of objects of possible

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[240] Kant, I., 1787, "*The Critique of Pure Reason: Preface To The Second Edition*", Translated By J. M. D. Meiklejohn, Retrieved 2003<*http://www.encarta.msn. com/*>
[241] *Ibid.*
[242] *Ibid.*
[243] *Ibid*.

experience. Objects of themselves[244] have *no existence,* and *space* and *time* exist only as part of the mind; where *intuitions* by which perceptions are measured and judged.

Kant[245] then states that a number of a priori concepts, which he called *categories*, exist. This *category* falls into four groups: those concerning *quantity* are *unity*, *plurality,* and *totality*; those concerning *quality* are *reality, negation,* and *limitation*; those concerning *relation* are *substance-and-accident, cause-and-effect,* and *reciprocity*; and those concerning *modality* are *possibility, existence,* and *necessity.* Kant's *transcendental method*[246] has permitted him to reveal the a priori components of *sensations* and the a priori *concepts*. There are *a priori judgments* that must necessarily govern all appearances of objects; these *judgments* are a function of the table of *categories' role* in determining all possible judgments. *Judgment* is the fundamental action of thinking. It is the process of conceptual unification of representations. Determining thought must be *judgmental* in form.

Concepts[247] are the result of judgments unifying further concepts; but this cannot be an infinitely regressing process. Certain concepts are basic to judgment and not themselves the product of prior judgments; these are the *categories* of the *pure* concepts. Therefore, the *categories* are necessary conditions of judging i.e. necessary conditions of thought. We can determine which concepts are the *pure* ones by considering the nature of judgment. *Judgments* can be viewed as unity functions for representations. Different forms of judgment will unify representations in different ways. *Understanding*[248] is the faculty of knowledge and the first pure knowledge of understanding is the principle of original *synthetic* unity of apperception; it is an objective condition of knowledge.

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[244] ….., *"Kant"* Retrieved 2004 <*http:// www.encarta.msn. com/*>
[245] *Ibid*.
[246] Wallis, S.F, 2004, *Immanuel Kant (1724-1804),* New York: Media & Communication, The European Graduate School. Retreived 2004 <http://www.uni.scrf.ucam.org/ccps/Kantian.pdf>
[247] Kant in Wallis, S.F, 2004, *Immanuel Kant (1724-1804),* New York: Media & Communication, The European Graduate School. Retreived 2004 <*http://www.uni.scrf.ucam.org/ccps/Kantian.pdf*>
[248] Kant, I., 1787, "*The Critique of Pure Reason: Preface To The Second Edition",* Translated By J. M. D. Meiklejohn, Retrieved 2003<*http://www.encarta.msn. com/*>

Kant[249] further claims that *transcendental* unity of apperception is how all the manifold given in an intuition is united in a concept of an object. It is *objective* and *subjective* unity of consciousness which is a determination of inner sense through which manifold is empirically given. Kant insists that *judgment* is the manner in which given *modes of knowledge* are brought to the objective unity of apperception. It indicates the objective unity of a given representation's relation to original apperception, and its necessary unity. Kant claims that the *representations* belong to one another in virtue of the necessary unity of apperception in the synthesis of intuition that accords to principles of the objective determination of all representations and only in this way does there arise from this relation a judgment which is objectively valid.

Kant[250] adds that all the *manifold* is determined in respect of one to the logical functions of judgment and is thereby brought into one consciousness; the *categories* are these functions of judgment. The *faculty of understanding* is a faculty for *synthesis* the unification of representations; the functioning of this faculty can be analyzed at two different levels. Corresponding to two different levels at which we may understand representations: *a general logical level* and *a transcendental level.* In terms of the former, *synthesis* results *analytic unity*; in terms of the latter, *synthesis* results *synthetic unity;* and the latter takes into account the difference between *pure* and *empirical* concepts. According to Kant, *analytic unity* is an analysis of a judgment at the *level of general logic* which indicates the formal relationship of concepts independently of their content; while *synthetic unity* refers to objectivity.

At the *transcendental level*, judgments[251] have *transcendental content*; that is, they are related to some objects; they are given to the understanding as being about something.

[249] *Ibid.*
[250] *Ibid.*
[251] *Ibid.*

This is more than a matter of having a certain logical form. In which the *Categories* takes play in a *judgmen*t, that *judgment* is a representation of an object. Kant says:

> If understanding as such is explicated as our power of rules, then the power of judgment is the ability to subsume under rules, i.e., to distinguish whether something does or does not fall under a given rule. [252]

The following stage[253] in Kant's project will be used to analyze the *formal* or *transcendental* features of experience that enable judgment. If there are any such features besides what the previous stages have identified, the cognitive power of judgment does have a *transcendental structure*.

Kant[254] argues that there are a number of principles that must necessarily be true of experience in order for judgment to be possible. Kant's analysis of judgment and the arguments for these principles are contained in his *Analytic of Principles*. According to Kant[255], the *sorts of judgments* consists of each of the following: some *quantity*, some *quality*, some *relation*, and some *modality*. Kant[256] states that any *intelligible thought* can be expressed in judgments of the above sorts; but, then it follows that any *thinkable experience* must be understood in these ways, and we are justified in projecting this entire way of thinking outside ourselves, as the inevitable structure of any possible experience. The *intuitions* and the *categories*[257] can be applied to make judgments about *experiences* and *perceptions*, but cannot, according to Kant, be applied to *abstract ideas* such as *freedom* and *existence* without leading to inconsistencies in the form of pairs of

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[252] Wallis, S.F, 2004, *Immanuel Kant (1724-1804),* New York: Media & Communication, The European Graduate School. Retreived 2004 <*http://www.uni.scrf.ucam.org/ccps/Kantian.pdf*>

[253] *Ibid.*

[254] *Ibid*.

[255] Kemerling, G., 2001, "*Kant: Synthetic A Priori Judgement.*". Retieved 2003 <*http://www. philoso phy pages.com referral/contact.htm*>

[256] *Ibid.*

[257] ….., "*Kant*" Retrieved 2004 <*http:// www.encarta.msn. com/*>

contradictory propositions, or "*antinomies*," in which both members of each pair can be proved true.

6. Analogies of Experience

Kant[258] elaborates that, in analogy, *experience* is possible only through the representation of a necessary connection of perceptions. Kant strives to prove this principle by exposing some arguments. First[259], *experience* is an *empirical cognition*; therefore it is a *synthesis of perceptions* i.e. a *synthesis* which is not itself contained in perception, but which contains the *synthetical* unity of the *manifold* of perception in a consciousness. This *unity* constitutes the essential of our cognition of objects of the senses, that is, of experience. Second[260], due to *apprehension* is only a placing together of the *manifold* of *empirical* intuition, in experience *our perceptions* come together contingently so that no character of necessity in their connection appears or can appear from the perceptions themselves,

Third[261], however, *experience* is cognition of objects by means of perceptions; it means that the relation of the existence of the *manifold* must be represented in *experience* not as it is put together in *time*, but as it is put objectively in *time*. Fourth[262], while *time* itself cannot be perceived, the determination of the existence of objects in *time* can only take place by means of their connection in *time* in general, consequently only by means of *a priori* connecting *conceptions*. As these *conceptions* always possess the character of necessity, *experience* is possible only by means of a representation of the necessary connection of perception.

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[258] Kant, I., 1787, "*The Critique of Pure Reason: Preface To The Second Edition*", Translated By J. M. D. Meiklejohn, Retrieved 2003<*http://www.encarta.msn. com/*>
[259] *Ibid.*
[260] *Ibid.*
[261] *Ibid.*
[262] *Ibid.*

Three *modus of time*[263] are *permanence*, *succession*, and *coexistence*; accordingly, there are three rules of all relations of *time* in *phenomena*, according to which the existence of every *phenomenon* is determined in respect of the unity of all *time*, and these antecede all experience and render it possible. The *general principle* of all three analogies[264] rests on the necessary unity of apperception in relation to all possible *empirical consciousness* at every *time*; consequently, as this unity lies *a priori* at the foundation of all mental operations, the principle rests on the *synthetical unity* of all *phenomena* according to their relation in *time*. According to Kant[265], for the *original apperceptio*n relates to our *internal sense* and indeed relates *a priori* to its form; it means that the relation of the *manifold* empirical consciousness in *time*. This *manifold* must be combined in original apperception according to relations of *time* i.e. a necessity imposed by the *a priori transcendental unity* of apperception.

All *empirical determinations* of time[266] must be subject to rules of the *general determination* of time; and the *analogies of experience* of which we are now about to treat must be rules of this nature. These *principles*[267] have this peculiarity, that is, they do not concern *phenomena* and the *synthesis* of the *empirical* intuition thereof, but merely the existence of *phenomena* and their relation to each other in regard to this existence. Now the *mode*[268] in which we apprehend a thing in a *phenomenon* can be determined *a priori* in such a manner that the *rule* of its *synthesis* can give or produce this *a priori intuition* in every *empirical* example. However, as Kant specifies, the existence of *phenomena* cannot

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[263] Kant, I., 1787, "*The Critique of Pure Reason: Preface To The Second Edition*", Translated By J. M. D. Meiklejohn, Retrieved 2003<*http://www.encarta.msn. com/*>
[264] *Ibid.*
[265] *Ibid.*
[266] *Ibid.*
[267] *Ibid.*
[268] *Ibid.*

be known *a priori* although we could arrive by this path at a conclusion of the fact of some existence.

We[269] could not cognize the *existence* determinately; it means that we should be incapable of anticipating in what respect the *empirical intuition* of it would be distinguishable from that of others. An *analogy of experience*[270] is, therefore, only *a rule* according to which *unity of experience* must arise out of perceptions in respect to objects not as *a constitutive,* but merely as *a regulative principle.* The same holds good of the *postulates of empirical thought* in general, which relates to the *synthesis* of *mere intuition* which concerns the form of *phenomena*, relates to the *synthesis of perception* which concerns the matter of *phenomena*, and relates to the *synthesis of experience* which concerns the relation of these perceptions.

a. First Analogy

In the "*Principle of Permanence of Substance"*, Kant, 1787, exposes that in all change of appearances *substance* is permanent; its *quantum* in nature is neither increased nor diminished. This *principle*[271] says that *all appearances* are in *time*. *Time* is the *substratum* in which *coexistence* or *succession* can be represented. *Time*[272] itself cannot be perceived; therefore, there must be in the objects perceived the substratum which represents *time* in general. Kant[273] mentions that the *substratum* of all real is substance; it is the permanent in relation to which alone all time-relations of appearances can be determined. In this "*First Analogy"*, Kant characterizes *substance* as "*something which can exist as subject and never as mere predicate."*

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[269] *Ibid*.
[270] *Ibid.*
[271] *Ibid*.
[272] *Ibid.*
[273] *Ibid.*

Substance[274] would mean simply a something which can be thought only as *subject*, never as a *predicate* of something else. It can exist as *subject* only, and not as a mere determination of other things. Our apprehension of the *manifold* in a *phenomenon* is always *successive* and consequently always changing. Without the *permanent*[275], then, no *relation* in *time* is possible. *Time* in itself is not an object of perception; consequently the *permanent* in *phenomena* must be regarded as the *substratum* of all determination of *time* and as the condition of the possibility of all *synthetical unity* of perceptions, that is, of experience. All *existence* and all *change* in *time* can only be regarded as a *mode* in the *existence* of that which abides unchangeably.

In all *phenomena*[276], the *permanent* is the object in itself, that is, the *substance* or *phenomenon*; but all that *changes* belongs only to the *mode of the existence* of this *substance* or *substances*. If[277] in the *phenomenon* which we call *substance* is to be the *proper substratum* of all determination of *time*, it follows that *all existences* in past as well as in future *time*, must be determinable by means of it alone. Accordingly, we are entitled to apply the term *substance* to a *phenomenon*, a notion which the word *permanence* does not fully express, only because we suppose its *existence* in all *time* as it seems rather to be referable to future time.

Change[278] is a mode of *existence* which follows another mode of existence of the same object; hence all changes is permanent, and only the condition there of changes. Since this *mutation* affects only determinations, which can have a beginning or an end, we may say that employing an expression which seems somewhat paradoxical that is only the *permanent substance* is subject to *change*. The *mutable* suffers *no change*, but rather

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[274] *Ibid*.
[275] *Ibid*.
[276] *Ibid*.
[277] *Ibid*.
[278] *Ibid*.

alternation, that is, when certain determinations cease, others begin. *Substances*[279] are the *substratum* of all determinations of *time*. The beginning of some *substances* and the ceasing of others would utterly do away with the only condition of the *empirical unity* of *time*. In this case *phenomena* would relate to two different *time*s, in which, side by side, *existence* would pass. For there is only one *time*[280] in which all different *time*s must be placed not as *coexistent* but as *successive*; accordingly, *permanence* is a *necessary condition* under which alone *phenomena*, as things or objects, are determinable in a *possible experience.*

b. Second Analogy

In the *"Second Analogy*", Kant[281] exposes that all *alterations* take place in conformity with the *law of the connection* of *cause* and *effect.* Kant proves that the *preceding principle* implies that all appearances of *succession* in *time* are *alterations* i.e. *not coming-to-be*; those *appearances* follow one another and connects two perceptions and thus this is a *synthetic faculty* of imagination. Kant[282] finds that the objective of *relation* of appearance of *succession* is not determined through *perception*. In order that this *relation* is known as determined, it must be so thought that it is thereby determined as *necessary* which comes first; and, *necessity* can only come from a *pure concept* of understanding; and thus, in this case, it is *cause* and *effect.* Further, Kant[283] sums up that the *apprehension* of the *manifold* of *appearance* is always *successive*. *Appearances,* simply in virtue of being *representations*, are not in any way distinct from their *apprehension* because we do not know if the parts of the object follow one another.

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[279] *Ibid.*
[280] *Ibid.*
[281] Kant, I., 1787, "*The Critique of Pure Reason: Preface To The Second Edition*", Translated By J. M. D. Meiklejohn, Retrieved 2003<*http://www.encarta.msn. com/*>
[282] *Ibid.*
[283] *Ibid.*

There is a *subjective succession*[284] e.g. of looking at a house top to bottom or left to right, as an *arbitrary succession*; while *objective succession* can be such an *order* in the *manifold* of *appearance* according to a rule that happens as an applies to *events*. *Appearance*[285] never goes backwards to some preceding *time*, but it does stand in relation to some preceding *time*; there must lie in that which precedes an *event* i.e. the condition of a rule according to which this *event* necessarily follows. Therefore, according to Kant, the *event,* as conditioned, thus affords reliable evidence of some condition; this condition is what determines the *event.* Kant[286] says that we have to show that we never ascribe *succession* to the object; when we perceive that something happens this representation contains the consciousness that there is something preceding.

Only by *reference*[287] to what preceded does the *appearance* acquire its *time relation*. The rule is that the condition under which an *event* necessarily follows lies in what precedes the *event*, called *the principle of sufficient reason*. It is the *ground of possible experience* in which the relation of *cause* to *effect* is the condition of the *objective validity* of our *empirical judgments*. Kant[288] notes that although *phenomena* are not *things in themselves* and nevertheless the only thing given to us to cognize, it is his duty to show what sort of *connection* in *time* belongs to the *manifold* in *phenomena themselves,* while the representation of this *manifold* is always *successive*. Accordingly, when we know in *experience* that something happens, we always presuppose that, in conformity with a rule, something precedes. He emphasizes that, in reference to a rule to which *phenomena* are determined in their *sequences,* we can make our *subjective synthesis* objective, and it is only under this *presupposition* that even the experience of an *event* is possible.

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[284] *Ibid*.
[285] *Ibid*.
[286] *Ibid*.
[287] *Ibid*.
[288] *Ibid*.

Kant[289] says that we have *representations* within us in which we should be conscious. Widely extended, accurate, and thorough going this consciousness may be, these *representations* are still nothing more than *representations*, that is, *internal determinations* of the mind in this or that relation of *time*. For *all experiences* and the *possibility of experience*[290], *understanding* is indispensable. The first step which it takes in this sphere is not to render clearly the *representation* of objects, but to render the *representation* of an object in general be possible; it does this by applying the order of *time* to *phenomena*, and their *existence*. All *empirical cognition*[291] belongs to the *synthesis* of the *manifold* by the *imagination* i.e. a *synthesis* which is always *successive* in which the *representation* always follow one another.

The *order of succession*[292] in *imagination* is not determined, and the *series of successive* representations may be taken retrogressively as well as progressively. If this *synthesis* is a *synthesis of apprehension*, then the *order* is determined in the object. There[293] is an *order of successive synthesis* which determines an object in which something necessarily *precedes*, and when this is posited, something else necessarily *follows*. The *relation of phenomena*[294] is necessarily determined in *time* by something *antecedes*, in other words, in conformity with a *rule.* The relation of *cause* and *effect* is the condition of the *objective validity* of our *empirical judgments* in regard to the sequence of perceptions of their *empirical truth* i.e. their experiences. The principle of the relation of *causality* in the *succession* of *phenomena* is therefore *valid* for all objects of experience because it is itself the *ground* of the possibility of experience.

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[289] *Ibid*.
[290] *Ibid*.
[291] *Ibid*.
[292] *Ibid*.
[293] *Ibid*.
[294] *Ibid*.

c. Third Analogy

In the "*Third Analogy"*, Kant[295] delivers the principle that *all substances*, in so far as they can be perceived to *coexist* in *space*, are in thorough going *reciprocity*. Kant strives to prove this principle with the following arguments: First[296], *things* are *coexistent* when in *empirical intuition,* the perceptions of them can follow upon one another *reciprocally*. Second[297], we cannot assume that because *things* are set in the same *time,* their *perceptions* can follow *reciprocally* in which *influence* is the relation of *substances* contains the *ground* of the determinations of another. The *community* or *reciprocity* is the *relation of substances* where each contains the *ground* of the determinations in the other.

Third[298], we know *two substances* in the same *time* when the *order* in the *synthesis of apprehension* of the *manifold* is a matter of indifference. Fourth[299], if *each* is completely *isolated, coexistence* would not be *a possible perception;* therefore, there must be something through which *A determines for B* and vice versa in which its position is in *time* and the *cause* of another determines the *position* of the other in *time*. It is necessary[300] that the *substances* stand immediately in *dynamical community* if their *coexistence* is to be known in any *possible experience*. *Things*[301] are *coexistent* when in *empirical intuition* the perception of the one can follow upon the perception of the other or which cannot occur in the *succession* of *phenomena*. C*oexistence* is the existence of the *manifold* in the same *time*, however *time* it-self is not an object of perception. Therefore we cannot conclude from

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[295] Kant, I., 1787, "*The Critique of Pure Reason: Preface To The Second Edition*", Translated By J. M. D. Meiklejohn, Retrieved 2003<*http://www.encarta.msn. com/*>
[296] *Ibid.*
[297] *Ibid.*
[298] *Ibid*.
[299] *Ibid.*
[300] *Ibid.*
[301] *Ibid.*

the fact that *things* are placed in the same *time*; while the perception of these *things* can follow each other reciprocally.

A *conception*[302] of the understanding or *category* of the *reciprocal sequence* of the determinations of *phenomena* is requisite to justify that the *reciprocal succession* of perceptions has its *foundation* in the object and to enable us to represent *coexistence* as objective. The *relation of substances,* in which the one contains determinations the ground of the other substance, is the relation of influence. When this influence is reciprocal, it is the relation of community or reciprocity. Consequently, the *coexistence*[303] of substances in *space* cannot be cognized in experience except that under the precondition of their reciprocal action. This is therefore the condition of the possibility of *things* themselves is an objects of experience. *Things* are *coexistent* in so far as they exist in one and the same *time*; but how can we know that they exist in one and the same *time*?

Every *substance*[304] must contain the *causality* of certain determinations in another *substance*, and at the same time the *effects of the causality* of the other in *itself.* If *coexistence* is to be cognized in any possible experience, s*ubstances* must stand in *dynamical community* with each other; however, it would itself be impossible if it is cognized without experiences of objects. Consequently, it[305] is absolutely necessary that *all substances* in the world of *phenomena*, in so far as they are *coexistent*, stand in a relation of *complete community* of reciprocal action to each other. Kant[306] finds *three dynamical relations* from which all others spring: *inherence*, *consequence*, and *composition;* these, then, are called *three analogies of experience.*

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[302] *Ibid*.
[303] *Ibid.*
[304] *Ibid*.
[305] *Ibid.*
[306] *Ibid.*

According to Kant[307], they are nothing more than *principles of the determination* of the *existence of phenomena* in *time*. Three *modi* of *determination*s covers *the relation to time itsel*f as a quantity, *the relation in time as a series or succession,* and *the relation in time as the complex of all existence*. Kant[308] claims that this *unity of determination* in regard to *time* is thoroughly *dynamical*. It says that *time* is not considered as *experience* determines immediately to every *existence* of its position because it is impossible that *absolute time* is not an object of perception in which *phenomena* can be connected with each other.

Question:

1. Explain the pure concept of understanding
2. Explain the noumena
3. Explain the phenomena
4. Explain the principle of analogy

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[307] *Ibid.*
[308] *Ibid.*

CHAPTER FIVE

SYSTEM OF A PRIORI CONCEPTS AND SYNTHETIC A PRIORI PRINCIPLES

Kompetensi Dasar Mahasiswa:

Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap persoalan-persoalan pokok dalam Sistem Sintetik A Priori

Materi Pokok :

Persoalan-persoalan Pokok Sistem Sintetik A Priori

Strategi Perkuliahan:

Ekspositori, diskusi, presentasi, refleksi

Sumber Bahan:

Pilih yang sesuai

Kant's[309] distinction between the *regulative* and *constitutive* uses of the understanding, a kind of dichotomous gap, reappears between the faculties of *reason* and *intuition*. Kant[310] justifies the *validity* of this distinction in two series of arguments where he also distinguishes between two different *regulative* uses of reason. The grounds[311] of these distinctions seem to follow the structure of his three ways division of the logic into *judgment, understanding* and *reason.* Each of these three activities has correlation in the *logical syllogism. Understanding*[312] is that faculty by which we make rules and the *generator* of the *Major Premise* in a syllogism. *Judgment*[313] is that by which we bring *particulars* under a

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[309] Faller, M., 2003, "*Kant's Mathematical Mistake*", Retrieved 2004 <*http://polar.alas kapacific.edu/mfaller/ KntMth.PDF.*>
[310] *Ibid.*
[311] *Ibid.*
[312] *Ibid.*
[313] *Ibid.*

property or class; this is the source of the *Minor Premise*. *Reason* is that by which we tie the premises together with the conclusion.

1. Transcendental Logic in the Critique of Pure Reason

Kant[314] elaborates the idea of *transcendental logic* in the Second Part of *"Transcendental Doctrine of The Elements"* of the "*Critique of Pure Reason*". In this Part, there are four Sub Topic: *Logic in General*, *Transcendental Logic*, *Division of General Logic into Analytic and Dialectic,* and *Division of Transcendental Logic into Transcendental Analytic and Dialectic*. *Logic In General* consists of two fundamental sources of knowledge: *sensibility* i.e. the capacity to receive representations which consists of the *Science of Aesthetic* and *How objects are given to us*; and *understanding* i.e. the power of knowing an object through representations which consists of *The science of Logic* and *How an object is thought.* Kant claims that only through their union can knowledge arise.

According to Kant[315], there are two types of *logic*: *logic in general* contains absolutely necessary rules of thought viz. *the logic of elements;* and *logic of the special employment of the understanding* contains rules of correct thinking about certain kinds of objects viz. *the logic of a particular science*. *General logic* consists of *pure* i.e. an abstracts from all *empirical* conditions, hence it deals with mere forms of thought; and consists of *applied* i.e. an understanding under *subjective empirical conditions.* Kant[316] characterizes *transcendental logic* not as an abstract from the entire content of knowledge. It excludes only those modes of knowledge which have *empirical content* and treats the origin of modes in which we know objects. Further, Kant[317] claims that not every kinds of *a priori*

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[314] Kant, I., 1787, "*The Critique of Pure Reason: Preface To The Second Edition"*, Translated By J. M. D. Meiklejohn, Retrieved 2003<*http://www.encarta.msn. com/*>
[315] *Ibid*.
[316] *Ibid.*
[317] *Ibid.*

knowledge should be called transcendental; only that by which we know that certain representations can be employed or are possible *a priori*. *Space* is the knowledge that the representations are not *empirical* one.

Kant[318] divides *transcendental logic* into *transcendental analytic* and *dialectic*. He elaborates that *transcendental analytic* has two aspects: *logic* which deals with elements of *pure knowledge* yielded by understanding and *logic* in which no object can be thought. In transcendental *dialectic*, a misuse of *transcendental analytic* and *dialectic illusion* may happen. *Dialectic*[319] is concerned with the *fallacies* produced when *metaphysics* is extended beyond *possible experience*; while the *Analytic*, about secure metaphysics, is divided into the *Analytic of Concepts* and the *Analytic of Principles.*

Kant[320] distinguishes *the science of the laws of sensibility* i.e. *aesthetic* from *the science of the laws of the understanding* i.e. *logic. Logic* in its turn may be considered as *logic of the general* or *of the particular* use of the understanding. The *first* contains the *absolutely necessary laws of thought* without which no use what so ever of the understanding is possible. It gives *laws to the understanding* without regard to the difference of objects on which it may be employed. The *second* contains the *laws of correct thinking* upon a particular class of objects. In a *pure general logic*[321] we abstract all the *empirical conditions* under which the understanding is exercised. It has to do merely with *pure a priori principles*. It is a canon of understanding and reason but only in respect of the formal part of their use to be the content of what it may be *empirical* or *transcendental*.

According to Kant[322], *in a pure general logic* we must always bear in mind two rules. First, *as general logic*, it makes abstraction of all content of the cognition of the

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[318] *Ibid*.
[319] *Ibid.*
[320] Ibid.
[321] *Ibid.*
[322] *Ibid*.

understanding and of the difference of objects. It has to do with nothing but the mere form of thought. Second, *as pure logic*, it has no *empirical principles* and consequently draws nothing from psychology which therefore has no influence on the canon of the understanding. It is *a demonstrated doctrine* in which everything in it must be certain completely *a priori*. In *an applied general logic* we direct the *laws of the use of the understanding* under the *subjective empirical conditions* in which psychology teaches us. It is an *empirical principle* although at the same *time*, it is in so far general, that it applies to the exercise of the understanding, without regard to the difference of objects.

Applied logic[323] is a representation of the understanding and of the rules of its necessary employment *in concreto* under the accidental conditions of the subject which may either hinder or promote this employment in which they are all given only *empirically*. Thus *applied logic*[324] treats of *attention, its impediments* and *consequences* of the origin of error, of the state of doubt, hesitation, conviction, etc. It relates *pure general logic* in the same way that *pure morality*. It contains only the necessary *moral laws of a free will,* is related to *practical ethics*. It considers these laws under all the impediments of feelings, inclinations, and passions to which peoples are more or less subjected. It can never furnish us with a true and demonstrated science because it, as well as applied logic, requires *empirical* and *psychological principles.*

With regard to our cognition in respect of its mere form, it[325] is equally manifest that *logic* exhibits the *universal* and *necessary laws of the understanding* and must in these very laws present us with criteria of truth. Whatever contradicts these *rules* is false because the *understanding* is made to contradict its own *universal laws of thought* i.e. contradict to itself.

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[323] *Ibid*.
[324] *Ibid.*
[325] *Ibid*.

These criteria[326], however, apply solely to the form of truth, that is, of thought in general, and in so far they are perfectly accurate, yet not sufficient. Although *cognition* may be perfectly accurate as to logical form or not *self-contradictory*, it is not withstanding quite possible that it may not stand in agreement with its object. Consequently, the merely *logical criterion of truth,* namely, the accordance of cognition with the *universal* and *formal laws of understanding* and *reason*, is nothing more than the *conditio sine qua non* or negative condition of all truth.

In the expectation[327] that there may be *mathematical conceptions* which relate *a priori* to objects, not as *pure* or *sensuous intuitions,* but merely as *acts* of *pure thought*, we form the *idea* of *a science of pure understanding* and *rational cognition* by cogitating objects entirely *a priori*. This kind of *science*[328] should determine the *origin*, the *extent,* and the *objective validity* of mathematical cognitions and must be called *transcendental logic.* Like in *general logic*[329], the *transcendental logic* has to do with the l*aws of understanding* and *reason* in relation to *empirical* as well as *pure rational cognitions* without distinction, but concerns itself with these only in an *a priori* relation to objects. In *transcendental logic*[330] we isolate the *understanding* and select from our *cognition* merely that part of thought which has its origin in the *understanding* alone.

Understanding and *judgment*[331] accordingly possess in *transcendental logic* a canon of *objectively valid*, *true exercise*, and *is comprehended* in the analytical department of that logic. However, *reason,* in her endeavors to arrive by *a priori* means at some true statement concerning objects and to extend cognition beyond the bounds of possible experience, is altogether *dialectic.* Its illusory assertions cannot be constructed into a canon such as an

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[326] *Ibid.*
[327] *Ibid.*
[328] *Ibid.*
[329] *Ibid.*
[330] *Ibid.*
[331] *Ibid.*

analytic ought to contain. *Logical illusion*[332], which consists merely in the imitation of the form of reason, arises entirely from a want of due attention to *logical rules. Transcendental dialectic*[333] will therefore content itself by exposing the *illusory appearance* in *transcendental judgments* and guarding us against it; but to make it, as in the case of *logical illusion,* entirely disappear and cease to be illusion is utterly beyond its power.

There[334] is a merely *formal logical use,* in which it makes abstraction of all content of cognition; but there is also a *real use*, in as much as it contains in itself the source of certain conceptions and principles, which it does not borrow either from the senses or from the understanding. As a division of reason into *a logical* and *a transcendental faculty* presents itself here, it becomes necessary to seek for *a higher conception* of this source of cognition which shall comprehend both conceptions. Here[335] we may expect, according to the *analogy of the conceptions of the understanding*, that the *logical conception* will give us the key to the *transcendental*, and that the *table of the functions* of the former will present us with the clue to the *conceptions of mathematical reason.*

2. The Method of Discovering the Concepts of the *Pure* Understanding

In the "*Critique of Pure Reason"*, Kant (1787) addresses the challenge of subsuming *particular sensations* under *general categories* in the *Schematism* section. Kant argues that *Transcendental Schemata* allow us to identify the homogeneous features picked out by concepts from the heterogeneous content of our sensations. Therefore, he indicates that *judgment* is only possible if the mind can recognize the components in the diverse and disorganized data of sense that make those sensations an instance of a concept or

[332] *Ibid.*
[333] *Ibid.*
[334] *Ibid.*
[335] *Ibid.*

concepts. Further, Kant argues that the *necessary conformity* of objects to *natural law* arises from the mind. Kant's *transcendental method* has permitted us to reveal the *a priori* components of sensations i.e. the *a priori concepts*. There are *a priori judgments* that must necessarily govern all appearances of objects. These *judgments* are a function of the *Table of Categories* in determining all *possible judgments.*

The continuity of nature[336] is also reflected in the *dynamical categories*, which are divided into those of *relation* and those of *modality*. The *relational* categories are *substance-accident, cause-effect,* and *agent-patient.* In each case, the corresponding principle is one of continuity. Kant[337] held that the only *change* occurred is a *change* in the state of an existing *thing*. Thus, there are no discontinuities of existence in nature, no new *things* coming to be, and no existing *things* passing away. All *change* is bound by *laws of nature,* which precludes the discontinuity that would result if *change* were random.

Following (Figure 13) is the *schematized of categories* which is summarized by Kant[338]:

Categories of the Understanding *As to: Quantity - Quality - Relation – Modality*			
Unity (Measure)	*Reality*	*Substance*	*Possibility*
Plurality (Magnitude)	*Negation*	*Cause*	*Existence*
Totality (Whole)	*Limitation*	*Community*	*Necessity*

Figure 13: Categories of Understanding

[336] Kant in *"Kant"* Retrieved 2004 <h**ttp://** www.encarta.msn. *com/*>
[337] *Ibid.*
[338] *Ibid.*

Since *individual images*[339] are perfectly separable as they occur within the *sensory manifold,* connections among them can be drawn only by the knowing subject in which the principles of connection are to find. As in *mathematics,* so in *science* the *synthetic a priori judgments* must derive from the structure of the understanding itself. *Transcendental illusion*[340] is the result of applying the *understanding* and *sensibility* beyond their limits. Although the *objective rules* may be the same in each case, the *subjective idea of causal connection* can lead to different deductions.

3. The Legitimate and Illegitimate Use of the Categories

Kant[341] argues that in the sections titled the *Axioms, Anticipations, Analogies, and Postulates,* there are *a priori judgments* that must necessarily govern all appearances of objects. These *judgments* are a function of the *Table of Categories* in determining all possible judgments. *Axioms of Intuition* states that all intuitions are extensive magnitudes. *Anticipations of Perception* states that in all appearances the real that is an object of sensation has intensive magnitude, i.e., a degree. *Analogies of Experience* states that: a. in all variations by appearances substance is permanent, and its quantum in nature is neither increased nor decreased; b. all changes occur according to the law of the connection of cause and effect; and c. all substances, insofar as they can be perceived in space as simultaneous, are in thoroughgoing interaction. *Postulates of Empirical Thought* states: a. what agrees with the formal conditions of experience is possible; b. what coheres with the material conditions of experience is actual; and that whose coherence with the actual is determined according to universal conditions of experience is necessary.

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[339] Kant in Kemerling, G., 2001, "*Kant: Synthetic A Priori Judgement.*". Retieved 2003 <*http://www.philosophypages.com referral/contact.htm*>
[340] Kan in Meibos, A., 1998, "*Intro to Philosophy: Kant and a priori Synthetic Judgments*", Prof. Arts Notes for PHIL 251 Retrieved 2004 <*http://www.icecavern.net/~qirien/punkus/index.html*>
[341] In Wallis, S.F, 2004, *Immanuel Kant (1724-1804),* New York: Media & Communication, The European Graduate School. Retreived 2004 <*http://www.uni.scrf.ucam.org/ccps/Kantian.pdf*>

Question:

1. Explain the concept of a priori
2. Explain the concept of a posteriori
3. Explain the concept of synthetic
4. Explain the concept of analytic

CHAPTER SIX
SPACE AND TIME

Kompetensi Dasar Mahasiswa:
Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap persoalan-persoalan pokok tentang Ruang dan Waktu

Materi Pokok :
Persoalan-persoalan Pokok tentang Ruang dan Waktu

Strategi Perkuliahan:
Ekspositori, diskusi, presentasi, refleksi

Sumber Bahan:
Pilih yang sesuai

Of the space and time, Kant concerns them with their metaphysical exposition and their relation to subjective conditions of sensation. According to Kant[342], a *pure* concept of space warrants and constrains intuitions of finite regions of *space*; that is, an *a priori* conceptual representation of *space* provides *a governing principle* for all spatial constructions, which is necessary for mathematical demonstration as Kant understood (Shabel, L.). Kant[343] notes that the *aesthetic* means to constitute and begin with an investigation of *space*. The concept[344] of *space* would be indistinguishable from the general concept of spaces in general. According to Kant, such a general concept itself rests on limitations of *space* and cannot itself be the source of the boundlessness of *space*.

[342] *Ibid.*
[343] *Ibid.*
[344] Shabel, l., 2003, "*Reflections on Kant's concept (and intuition) of space*", Studies In History and Philosophy of Science Part A Volume 34, Issue 1 Retreived 2003, <*http://www. sciencedirect. com/science?*>

Thus, an exposition of such a general concept of spaces could not be expected to satisfy Kant's goals in the *Transcendental Aesthetic* (Shabel, L.). Kant[345] identifies that a concept of *space* is strictly identical neither to a general concept of *space*s, nor to any particular intuition. Kant[346] admits that *space* could not be an *empirical concept.*

According to Kant[347], concepts are not singular, nor can they contain infinitely many parts; thus, *space* is represented in intuition and it seems equally impossible to intuit a single infinitely large object. Therefore, according to Kant's, this would require that we be able to form an immediate (unmediated) representation of an infinite spatial magnitude, that we grasp its infinitude in a single `glance', as it were (Shabel, L.). So, Kant[348] uses the *Metaphysical Exposition*, at least in part, to describe the *pure* spatial intuition that underlies any and all geometric procedures, but he does not use properly geometric procedures to describe that intuition. While cognition of the `*axioms*' of geometry depends, in some sense, on our having a capacity for *pure* spatial intuition, that capacity cannot itself be described as a capacity for geometric reasoning. So, our capacity for *pure* spatial intuition[349], described in the *Metaphysical Exposition*, is pre-geometric in the sense that it is independent of and presupposed by Euclidean reasoning.

Kant in Ross, K.L. (2001) proposes that *space* and *time* do not really exist outside of us but are *forms of intuition* i.e. conditions of perception imposed by our own minds. This enables Kant to reconcile Newton and Leibniz. Kant agrees with Newton that *space* is absolute and real for objects in experience, i.e. for *phenomena*l objects open to science. However, Kant also agrees with Leibniz that *space* is really nothing in terms of objects as

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[345] Gottfried, P., 1987, *"Form of Intuition: Kantian Time And Space Reconsidered",* The World & I: Issue Date: AUGUST 1987 Volume:02 Page: 689. Retrieved 2004 *<http://www.worldandi. com/public/1987/ august/copyright.asp>*
[346] *Ibid*,
[347] *Ibid*.
[348] *Ibid*.
[349] *Ibid*.

they exist apart from us, i.e. with *things in themselves*. The bulk of Kant's exposition on *time* and *space* in relation to sensory perception can be found in the opening pages of The *Critique of Pure Reason* (1781) (Gottfried, P., 1987). In the first part of the *Critique*, the "*Transcendental Aesthetic*," Kant treats of *time* and *space* as the *a priori* condition for cognition. Kant examines *time* and *space* as universal forms of intuition that help render sensory impressions intelligible to the human mind.

Kant delivers his explanation to clarify distinction between *appearance* and *illusion*, a confused representation of reality. According to Kant, in *space* and *time*, *intuition* represents both *external objects* and *the self-intuition of the mind*. It affects our senses. *Appearance objects* are always seen as *truly given* providing that their situation depends upon the subject's mode of intuition and that the object as appearance is distinguished from an object in itself. According to Kant, we need not to say that body simply seems to be outside of us when we assert that the quality of *space* and *time* lies in our mode of intuition and not in objects in themselves. [350]

Question:

1. Explain the concept of Time
2. Explain the concept of Space

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[350] Gottfried, P., 1987, *"Form of Intuition: Kantian Time And Space Reconsidered"*, The World & I: Issue Date: AUGUST 1987 Volume:02 Page: 689. Retrieved 2004 <*http://www.worldandi. com/public/1987/ august/copyright.asp*>

CHAPTER SEVEN
THEORY OF JUDGMENT

Kompetensi Dasar Mahasiswa:

Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap persoalan-persoalan pokok dalam Teori Pengambilan Keputusan

Materi Pokok :

Persoalan-persoalan Pokok Teori Pengambilan Keputusan

Strategi Perkuliahan:

Ekspositori, diskusi, presentasi, refleksi

Sumber Bahan:

Pilih yang sesuai

Kant[351] elaborates that *judgments* are complex conscious cognitions, that: 1) refer to objects either directly (via intuitions) or indirectly (via concepts), 2) include concepts that are predicated either of those objects or of other constituent concepts, 3) exemplify *pure* logical concepts and enter into inferences according to *pure* logical laws, 4) essentially involve both the following of rules and the application of rules to the objects picked out by intuitions, 5) express true or false propositions, 6) mediate the formation of beliefs, and 7) are unified and self-conscious. Correspondingly[352], a Kantian cognitive faculty is innate in the three fold senses, that: 1) it is intrinsic to the mind, hence a necessary part of the nature

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[351] Hanna, R., 2004, "*Kant's Theory of Judgment*", Stanford Encyclopedia of Philosophy, Retreived 2004, <*http://plato.stanford.edu/cgi-bin/encyclopedia/ archinfo.cgi?entry=*
kant-judgment>

[352] Kant in Hanna, R., 2004, "*Kant's Theory of Judgment*", Stanford Encyclopedia of Philosophy, Retreived 2004, <*http://plato.stanford.edu/cgi-bin/encyclopedia/ archinfo.cgi?entry=*
kant-judgment>

of the rational animal possessing that faculty, 2) it contains internal structures that are underdetermined by sensory impressions — which is the same as their being *a priori*, and 3) it automatically systematically synthesizes those sensory inputs according to special rules that directly reflect the internal structures of the faculty, thereby generating its correspondingly-structured outputs.

Understanding and *sensibility*[353] are both sub-served by the faculty of *imagination* (*Einbildungskraft*), which when taken generically is the source or engine of all sorts of *synthesis*, but which when taken as a dedicated to *task-sensitive cognitive faculty*, more specifically generates: 1) the *spatial* and *temporal* forms of intuition, 2) novel mental imagery in conscious sensory states, 3) reproductive imagery or memories, and 4) *schemata*, which are supplementary rules for interpreting general conceptual rules in terms of more specific figural *(spatio-temporal)* forms and sensory images. According to Kant[354], *judgment* is the mediate cognition of an object and hence it is *the representation of a representation* of it. In every *judgment* there is a concept that holds of many (representations), and that among this many also comprehends a given representation, which is then immediately referred to the object.

All *judgments*[355] are functions of unity among our representations, since instead of an immediate representation a higher one, which comprehends this and other representations under itself, is used for the cognition of the object, and many possible cognitions are hereby drawn together into one. A *judgment*[356] is nothing more than the way to bring given cognitions to the objective unity of apperception. Kant's questions the ground

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[353] *Ibid.*
[354] Hanna, R., 2004, "*Kant's Theory of Judgment*", Stanford Encyclopedia of Philosophy, Retreived 2004, *<http://plato.stanford.edu/cgi-bin/encyclopedia/ archinfo.cgi?entry=*
kant-judgment>
[355] Kant in Hanna, R., 2004, "*Kant's Theory of Judgment*", Stanford Encyclopedia of Philosophy, Retreived 2004, *<http://plato.stanford.edu/cgi-bin/encyclopedia/ archinfo.cgi?entry=kant-judgment>*
[356] *Ibid.*

of the reference of that in us which we call *representation* to the object that is the possibility of valid mental representations, is the fundamental topic of Kant's "*theory of cognition*". Kant[357] insists that justified true belief is *scientific knowing* which connects epistemology in Kant's sense directly with his conception of a *science* as a systematically unified body of cognitions based on *a priori* principles.

Kant[358] holds that *a belief* constitutes *scientific knowing* if and only if the *judgment* underlying that belief is not only *subjectively sufficient* for believing but is also *objectively sufficient* one, and coherent with a suitably wide set of other beliefs, and *also* true, although it still remains fallible. The *objective sufficiency* of a judgment for Kant[359] is the *inter-subjectively* rationally communicable conscious state of "*conviction"*, which is also the same as "*certainty"*. One of the most controversial, influential, and striking parts of *Kant's theory of judgment* is his multiple classification of judgments according to kinds of *logical form* and kinds of *semantic content.*

Indeed, the very importance of Kant's multiple classification of judgments[360] has sometimes led to the misconception that his theory of judgment will stand or fall according to the fate of, e.g., his *analytic-synthetic* distinction, or the fate of his doctrine of *synthetic a priori* judgments. The core[361] of *Kant's theory of judgment* consists in the central *thesis* and the priority of the proposition thesis, both of which can still hold even if some of his classifications of judgments are rejected. The *table of judgments*[362], in turn, captures a fundamental part of the science of *pure general logic*: *pure*, because it is *a priori*, necessary, and without any associated sensory content; *general*, because it is both *universal* and *essentially formal,* and thereby abstracts away from all specific objective representational contents and from the differences between particular represented objects;

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[357] *Ibid.*
[358] Kant, I., 1781, *"The Critique of Pure Reason: Transcendental Analytic, Book II, Analytic Of Principles"* Translated By J. M. D. Meiklejohn, Retrieved 2003 <*http://Www.Encarta.Msn. Com/*>*)*
[359] *Ibid.*
[360] *Ibid.*
[361] *Ibid.*
[362] *Ibid.*

and *logic* because, in addition to the table of judgments, it also systematically provides *normative cognitive rules* for the truth of judgments and for valid inference.

Kant's table of judgments[363] lays out an exhaustive list of the different possible logical forms of propositions under four major headings, each major heading containing three sub-kinds, as follows[364]:

1. Quantity of Judgments : *Universal, Particular, Singular*
2. Quality : *Affirmative, Negative, Infinite*
3. Relation : *Categorical, Hypothetical, Disjunctive*
4. Modality : *Problematic, Assertoric, Apodictic*.

For Kant[365], the *propositional content of a judgment* is more basic than its *logical form.* The propositional content of a judgment, in turn, can vary along at least three different dimensions: (1) its relation to *sensory content;* (2) its relation to the *truth-conditions of propositions*; and (3) its relation to the *conditions for objective validity.*

The notion of *cognitive content* for Kant[366] has two sharply distinct senses: 1) *intension,* which is *objective* and *representational* (semantic content); and 2) *sensory matter,* which is *subjective* and *non-representational*, reflecting only the immediate conscious response of the mind to the external impressions or inputs that trigger the operations of the faculty of *sensibility*. To be sure, for Kant[367], just as for the *Empiricists, all cognition* begins with the *raw data* of sensory impressions. But in a crucial departure from *Empiricism* and towards what might be called a *mitigated rationalism*, Kant[368] also holds that *not all cognition* arises from sensory impressions: so for him, a significant and unique

[363] *Ibid.*
[364] *Ibid.*
[365] Kant in Hanna, R., 2004, "*Kant's Theory of Judgment"*, Stanford Encyclopedia of Philosophy, Retreived 2004, <*http://plato.stanford.edu/cgi-bin/encyclopedia/ archinfo.cgi?entry=kant-judgment*>
[366] *Ibid.*
[367] *Ibid.*
[368] *Ibid.*

contribution to both the form and the objective representational content of cognition arises from *the innate spontaneous cognitive capacities.*

Applying the notions to judgments[369], it follows that a *judgment* is *a posteriori* if and only if either its logical form or its propositional content is strictly determined by sensory impressions; and a *judgment* is *a priori* if and only if neither its logical form nor its propositional content is strictly determined by sensory impressions and both are instead strictly determined by our *innate spontaneous cognitive faculties,* whether or not that cognition also contains sensory matter. Kant[370] also holds that a *judgment* is *a priori* if and only if it is *necessarily true.* This strong connection between *necessity* and *apriority* expresses: 1) Kant's view that the *contingency of a judgmen*t is bound up with the modal dependence of its semantic content on sensory impressions, i.e., it's aposteriority , 2) his view that necessity is equivalent with strict universality or *strenge Allgemeinheit*, which he defines in turn as a proposition's lack of any possible counterexamples or falsity-makers, and 3) his view that necessity entails truth.

Kant's distinction[371] between *analytic* and *synthetic judgments* is as: (1) *analyticity* is truth by virtue of linguistic meaning alone, exclusive of *empirical facts*, (2) *synthetic*ity is truth by virtue of *empirical* facts, and (3) the *necessary statement* vs. *contingent statement* distinction is *formally* and *materially* equivalent to the *analytic-synthetic* distinction. A *judgment*[372] is *analytic* if and only if its propositional content is necessarily true by virtue of necessary internal relations between its objectively valid conceptual microstructures or its conceptual comprehensions. A *proposition*[373] is *synthetic* if and only if its truth is *not* strictly determined by relations between its conceptual microstructures or conceptual

[369] *Ibid*.
[370] *Ibid.*
[371] *Ibid*.
[372] *Ibid.*
[373] *Ibid.*

comprehensions alone; and a *judgment* is *synthetically* true if and only if it is true and its denial does *not* logically entail a contradiction.

This[374] is not to say either that *synthetic judgments* do not contain any concepts or even that the conceptual components of a *synthetic judgment* are irrelevant to its meaning or truth but only to say that in a *synthetic judgment* it is the intuitional components that strictly determine its meaning and truth, not its conceptual components. In short, a *synthetic judgment* is *an intuition-based proposition.* Combining the *a priori-a posteriori* distinction with the analytic-*synthetic* distinction, Kant[375] derives four possible kinds of judgment: (1) *analytic* a *priori,* (2) *analytic a posteriori*, (3) *synthetic a priori*, and (4) *synthetic a posteriori*. By virtue of the fact that *analytic judgments* are *necessarily true*, and given Kant's thesis that *necessity entails apriority,* it follows that *all analytic judgment*s are *a priori* and that there is no such thing as an *analytic a posteriori judgment.* By contrast[376], *synthetic judgments* can be either *a priori* or *a posteriori*. *Synthetic a posteriori judgments* are *empirical* and *contingent* although they may vary widely to their degree of generality. *Synthetic a priori judgments,* by contrast, are *non-empirical and non-contingent judgments.*

Question:

Explain the various kinds of judgment.

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[375] *Ibid.*

[376] *Ibid.*

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SILABUS

Program Studi	:	**Pendidikan Matematika (S2)**
Mata Kuliah	:	Filsafat Ilmu
Jumlah Semester	:	2
Semester	:	Gasal
Dosen	:	Dr. Marsigit, MA ***http://powermathematics.blogspot.com***

I. Deskripsi Mata Kuliah

Mata Kuliah ini dimaksudkan untuk memberi kesempatan dan pelayanan kepada mahasiswa untuk membangun pemahaman dan teori tentang filsafat ilmu. Kajian perkuliahan meliputi: (1) Persoalan-persoalan Pokok dalam Pengembangan Ilmu (Matematika dan Pendidikan Matematika), (2) Karakteristik Ilmu (Matematika dan Pendidikan Matematika), (3) Obyek Ilmu (Matematika dan Pendidikan Matematika), (4) Metode Pengembangan Ilmu (Matematika dan Pendidikan Matematika), (5) Alat Pengembangan Ilmu (Matematika dan Pendidikan Matematika), (6) Sejarah Perkembangan Ilmu (Matematika dan Pendidikan Matematika), (7) Pre-Asumsi dan Asumsi Dasar Pengembangan Ilmu (Matematika dan Pendidikan Matematika), (8) Sumber-sumber dan Batas-batas Pengembangan Ilmu (Matematika dan Pendidikan Matematika), (9) Pembenaran Ilmu (Matematika dan Pendidikan Matematika), (10) Prinsip-prinsip Pengembangan Ilmu (Matematika dan Pendidikan Matematika), (11) Berbagai Aliran Pengembangan Ilmu (Matematika dan Pendidikan Matematika), (12) Ontologi Ilmu (Matematika dan Pendidikan Matematika), (13) Epistemologi Ilmu (Matematika dan Pendidikan Matematika), dan (14) Aksiologi Ilmu (Matematika dan Pendidikan Matematika), (15) Filsafat Matematika, (16) Filsafat Pendidikan Matematika

Dalam perkuliahan ini mahasiswa diberi kesempatan dan pelayanan untuk membangun pemahaman dan teori tentang filsafat ilmu melalui berbagai kegiatan meliputi: kegiatan ekspositori, diskusi, dan penugasan dosen agar dapat mengembangkan tesis-tesis pengembangan ilmu, mengembangkan anti tesis pengembangan ilmu, melakukan sintesis-sintesis untuk menghasilkan tesis-tesis

baru pengembangan ilmu, dan membangun struktur ontologi, epistemologi dan aksiologi filsafat ilmu pada umumnya, filsafat ilmu (matematika) dan filsafat ilmu (pendidikan matematika). Semua kegiatan tersebut dilakukan dan dikembangkan dalam kerangka pemahaman dan pengembangan jati diri manusia beserta ilmunya secara hermeneutikal.

II. Tujuan Mata Kuliah/ Kompetensi yang dikembangkan

Selama dan setelah mengikuti perkuliahan ini, mahasiswa diharapkan:

1. Memiliki motivasi dan keinginan yang tinggi disertai kesadaran akan pentingnya memahami dan mempelajari filsafat ilmu berdasarkan keyakinan dan pengalaman hidupnya.
2. Memiliki dan mengembangkan sikap atau perilaku yang menunjang serta sinkron dengan keinginannya mempelajari dan mengembangkan filsafat ilmu.
3. Memiliki dan mengembangkan pengetahuan dan mengetahui dan menggali sumber-sumber pengetahuan beserta obyek, alat dan metode pembenarannya.
4. Memiliki dan mengembangkan keterampilan untuk mengenali, menggali dan memproduksi tesis-tesis dan anti-tesisnya serta melakukan sintesis-sintesis untuk memproduksi tesis-tesis baru.
5. Memiliki dan mengembangkan pengalaman untuk merefleksikan diri dalam komunitas sosialnya perihal motivasi, sikap, pengetahuan dan keterampilannya mengembangkan ilmu, ilmu (matematika), dan ilmu (pendidikan matematika).

III. Strategi Perkuliahan dan Bentuk Kegiatan

A. Strategi Perkuliahan

Strategi perkuliahan dikembangkan dengan prinsip **belajar dapat dilakukan anywhere, anytime dan kontinu** (tidak dibatasi ruang dan waktu). Oleh karena itu strategi yang dikembangkan meliputi perkuliahan tatap muka dan perkuliahan Online. Perkuliahan tatap muka meliputi: 1. eksposisi dari dosen, 2. presentasi mahasiswa, 3. diskusi dosen mahasiswa, mahasiswa-mahasiswa, 4. mencari dan mengembangkan sumber belajar (internet dan referensi buku), 5. menyusun makalah untuk topik-topik terkait, 6. presentasi makalah, 7. balikan dosen

B. Bentuk Kegiatan:

1. Perkuliahan Tatap Muka

a. Tugas Mandiri Eksplorasi Sumber
b. Diskusi Kelas
c. Presentasi
d. Seminar
e. Ujian Sisipan
f. Ujian Akhir

2. Perkuliahan Online

Kuliah dan Tanya Jawab Online melalui **http://powermathematics.blogspot.com**

IV. Sumber Acuan

Utama:

1. Bolzano, B., 1810, "Appendix: On the Kantian Theory of the Construction of Concepts through Intuitions" in Ewald, W., 1996, "From Kant to Hilbert: A Source Book in the Foundations of Mathematics, Volume I", Oxford: Clarendon Press
2. Ernest, P., 1994, *Mathematics, Education and Philosophy: An International Perspective.* The Falmer Press: London.
3. Ernest, P., 2002, *What Is The Philosophy Of Mathematics Education?*
4. *Eves, H and Newsom, C.V., 1964, "An Introduction to the Foundation & Fundamental Concepts of Mathematics"*, New York: Holt, Rinehart and Winston
5. Ewald, W., 1996, "From Kant to Hilbert: A Source Book in the Foundations of Mathematics, Volume I", Oxford: Clarendon Press
6. Hers, R., 1997, What is Mathematics, Really?, London: Jonathan Cape
7. Kant, I., 1781, *Critic of Pure Reason*, Translatedby J.M.D. Meiklejohn
8. Mayer, F., 1951, "A History of Modern Philosophy", New York: American Book Company
9. Perry, R.B., 1912, Present Philosophical Tendencies: A Critical Survey of Naturalism Idealism Pragmatism and Realism Together with a Synopsis of the Pilosophy of William James, New York: Longmans Green and Co.

V. Penilaian

Penilaian meliputi kemampuan lisan, tulis dan portfolio dengan aspek-aspek meliputi:

1. Motivasi mempelajari dan mengembangkan filsafat ilmu (Matematika dan Pendidikan Matematika)
2. Sikap yang menunjang pengembangan filsafat ilmu(Matematika dan Pendidikan Matematika)
3. Pengetahuan aspek pengembangan filsafat ilmu (Matematika dan Pendidikan Matematika)
4. Keterampilan pengembangan filsafat ilmu (Matematika dan Pendidikan Matematika)
5. Pengalaman pengembangan filsafat ilmu (Matematika dan Pendidikan Matematika)

VI. Kegiatan Perkuliahan

Pertemuan ke	Kompetensi Dasar Mahasiswa:	Materi Pokok	Strategi Perkuliahan	Sumber Bahan
1.	Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap persoalan-persoalan pokok dalam	1. Persoalan-persoalan Pokok dalam Pengembangan Ilmu (Matematika dan Pendidikan	Ekspositori, diskusi, presentasi, refleksi	Pilih yang sesuai

	Pengembangan Ilmu(Matematika dan Pendidikan Matematika)	Matematika)		
2.	Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap karakteristik ilmu(Matematika dan Pendidikan Matematika)	2. Karakteristik Ilmu (Matematika dan Pendidikan Matematika)	Ekspositori, diskusi, presentasi, refleksi	Pilih yang sesuai
3.	Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap obyek ilmu(Matematika dan Pendidikan Matematika)	3. Obyek dan Metode Ilmu (Matematika dan Pendidikan Matematika)	Ekspositori, diskusi, presentasi, refleksi	Pilih yang sesuai
4.	Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap alat pengembangan ilmu(Matematika dan Pendidikan Matematika)	4. Alat Pengembangan Ilmu (Matematika dan Pendidikan Matematika)	Ekspositori, diskusi, presentasi, refleksi	Pilih yang sesuai
	Ujian Sisipan I			
5.	Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap sejarah perkembangan ilmu(Matematika dan Pendidikan Matematika)	5. Sejarah Perkembangan Ilmu (Matematika dan Pendidikan Matematika)	Ekspositori, diskusi, presentasi, refleksi	Pilih yang sesuai
6.	Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap pre-asumsi dan asumsi dasar pengembangan ilmu(Matematika dan Pendidikan Matematika)	6. Pre-Asumsi dan Asumsi Dasar Pengembangan Ilmu (Matematika dan Pendidikan Matematika)	Ekspositori, diskusi, presentasi, refleksi	Pilih yang sesuai
7.	Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap Sumber-sumber dan batas-batas pengembangan ilmu(Matematika dan Pendidikan Matematika)	7. Sumber-sumber dan Batas-batas Pengembangan Ilmu (Matematika dan Pendidikan Matematika)	Ekspositori, diskusi, presentasi, refleksi	Pilih yang sesuai
8.	Mengembangkan tesis, anti-tesis dan melakukan	8. Pembenaran dan Prinsip-prinsip	Ekspositori, diskusi,	Pilih yang

	sintesis terhadap pembenaran ilmu(Matematika dan Pendidikan Matematika)	Pengembangan Ilmu (Matematika dan Pendidikan Matematika)	presentasi, refleksi	sesuai
	Sisipan II			
9.	Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap ontologi ilmu	9. Ontologi Ilmu (Matematika dan Pendidikan Matematika)	Ekspositori, diskusi, presentasi, refleksi	Pilih yang sesuai
10.	Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap Berbagai Aliran Pengembangan Ilmu(Matematika dan Pendidikan Matematika)	10. Berbagai Aliran Pengembangan Ilmu (Matematika dan Pendidikan Matematika)	Ekspositori, diskusi, presentasi, refleksi	Pilih yang sesuai
11.	Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap epistemologi ilmu (Matematika dan Pendidikan Matematika)	11. Epistemologi Ilmu (Matematika dan Pendidikan Matematika)	Ekspositori, diskusi, presentasi, refleksi	Pilih yang sesuai
12.	Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap aksiologi ilmu(Matematika dan Pendidikan Matematika)	12. Aksiologi Ilmu (Matematika dan Pendidikan Matematika)	Ekspositori, diskusi, presentasi, refleksi	Pilih yang sesuai
13.	Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap filsafat ilmu (matematika)	13. Filsafat Matematika	Ekspositori, diskusi, presentasi, refleksi	Pilih yang sesuai
14.	Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap filsafat ilmu (pendidikan matematika)	14. Filsafat Pendidikan Matematika	Ekspositori, diskusi, presentasi, refleksi	Pilih yang sesuai
	UJIAN AKHIR			

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SILABUS

Program Studi	:	Pendidikan Sain (S2)
Mata Kuliah	:	Filsafat Ilmu
Jumlah Semester	:	2
Semester	:	Gasal
Dosen	:	Dr. Marsigit, MA *http://powermathematics.blogspot.com*

VII. Deskripsi Mata Kuliah

Mata Kuliah ini dimaksudkan untuk memberi kesempatan dan pelayanan kepada mahasiswa untuk membangun pemahaman dan teori tentang filsafat ilmu. Kajian perkuliahan meliputi: (1) Persoalan-persoalan Pokok dalam Pengembangan Ilmu (Sain dan Pendidikan Sain), (2) Karakteristik Ilmu (Sain dan Pendidikan Sain), (3) Obyek Ilmu (Sain dan Pendidikan Sain), (4) Metode Pengembangan Ilmu (Sain dan Pendidikan Sain), (5) Alat Pengembangan Ilmu (Sain dan Pendidikan Sain), (6) Sejarah Perkembangan Ilmu (Sain dan Pendidikan Sain), (7) Pre-Asumsi dan Asumsi Dasar Pengembangan Ilmu (Sain dan Pendidikan Sain), (8) Sumber-sumber dan Batas-batas Pengembangan Ilmu (Sain dan Pendidikan Sain), (9) Pembenaran Ilmu (Sain dan Pendidikan Sain), (10) Prinsip-prinsip Pengembangan Ilmu (Sain dan Pendidikan Sain), (11) Berbagai Aliran Pengembangan Ilmu (Sain dan Pendidikan Sain), (12) Ontologi Ilmu (Sain dan Pendidikan Sain), (13) Epistemologi Ilmu (Sain dan Pendidikan Sain), (14) Aksiologi Ilmu (Sain dan Pendidikan Sain), (15) Filsafat Sain, dan (16) Filsafat Pendidikan Sain

Dalam perkuliahan ini mahasiswa diberi kesempatan dan pelayanan untuk membangun pemahaman dan teori tentang filsafat ilmu melalui berbagai kegiatan meliputi: kegiatan ekspositori, diskusi, dan penugasan dosen agar dapat mengembangkan tesis-tesis pengembangan ilmu, mengembangkan anti tesis pengembangan ilmu, melakukan sintesis-sintesis untuk menghasilkan tesis-tesis baru pengembangan ilmu, dan membangun struktur ontologi, epistemologi dan aksiologi filsafat ilmu pada umumnya, filsafat ilmu (matematika) dan filsafat ilmu (pendidikan matematika). Semua kegiatan tersebut dilakukan dan dikembangkan dalam kerangka pemahaman dan pengembangan jati diri manusia beserta ilmunya secara hermeneutikal.

VIII. Tujuan Mata Kuliah/ Kompetensi yang dikembangkan

Selama dan setelah mengikuti perkuliahan ini, mahasiswa diharapkan:

6. Memiliki motivasi dan keinginan yang tinggi disertai kesadaran akan pentingnya memahami dan mempelajari filsafat ilmu (Sain dan Pendidikan Sain) berdasarkan keyakinan dan pengalaman hidupnya.
7. Memiliki dan mengembangkan sikap atau perilaku yang menunjang serta sinkron dengan keinginannya mempelajari dan mengembangkan filsafat ilmu (Sain dan Pendidikan Sain).
8. Memiliki dan mengembangkan pengetahuan dan mengetahui dan menggali sumber-sumber pengetahuan beserta obyek, alat dan metode pembenaran ilmu (Sain dan Pendidikan Sain)
9. Memiliki dan mengembangkan keterampilan untuk mengenali, menggali dan memproduksi tesis-tesis dan anti-tesisnya serta melakukan sintesis-sintesis untuk memproduksi tesis-tesis baru suatu ilmu (Sain dan Pendidikan Sain)
10. Memiliki dan mengembangkan pengalaman untuk merefleksikan diri dalam komunitas sosialnya perihal motivasi, sikap, pengetahuan dan keterampilannya mengembangkan Sain, dan Pendidikan Sain.

IX. Bentuk Kegiatan:

1. Perkuliahan Tatap Muka
2. Tugas Mandiri
3. Dsikusi
4. Presentasi
5. Seminar
6. Ujian Sisipan
7. Ujian Akhir

X. Strategi Perkuliahan

Strategi perkuliahan dikembangkan secara bervariasi meliputi: 1. eksposisi dari dosen, 2. presentasi mahasiswa, 3. diskusi dosen mahasiswa, mahasiswa-mahasiswa, 4. mencari dan mengembangkan sumber belajar (internet dan referensi buku), 5. menyusun makalah untuk topik-topik terkait, 6. presentasi makalah, 7. balikan dosen

XI. Sumber Acuan

Utama:

1. Kant, I., 1781, *Critic of Pure Reason*, Translatedby J.M.D. Meiklejohn
2. Mayer, F., 1951, "A History of Modern Philosophy", New York: American Book Company
3. Perry, R.B., 1912, Present Philosophical Tendencies: A Critical Survey of Naturalism Idealism Pragmatism and Realism Together with a Synopsis of the Pilosophy of William James, New York: Longmans Green and Co.
4. Ervin Laszlo, 2005, Science and the Akashic Field
5. Ervin Laszlo, 2006, The Chaos Point
6. Ervin Laszlo, 2003, The Connectivity Hypothesis
7. Mathieu Ricard & Trinh Xuan Thuan, 2003, The Quantum and the Lotus A Journey to the Frontiers Where Science and Buddhism Meet
8. Lynne McTaggart, 2003, The Field The Quest for the Secret Force of the Universe
9. Gary E. Schwartz, PhD & Linda G. S. Russek. PhD, 1999, The Living EnergyUniverse
10. A Fundamental Discovery that Transforms Science & Medicine
11. David Bohm, 1980, Wholeness and the Implicate Order

Tambahan:

1. Bonaccini, J.A., 2004, "*Concerning the Relationship Between Non-Spatiotemporality and Unknowability of Things in Themselves in Kant's Critique of Pure Reason*", Federal University of Rio Grande do Norte/National Research and Development Council, Brazil, <juan@cchla.ufrn.br >
2. Bowman, C., 2001, "*Kant and the Project of Enlightenment",* Department of Philosophy, University of Pennsylvania
3. Brook, A., 2004, "*Kant and Cognitive Science*, <http://www.teleskop.es/ ciencia/CienciaA03>

XII. Penilaian

Penilaian meliputi kemampuan lisan, tulis dan portfolio dengan aspek-aspek meliputi:

6. Motivasi mempelajari dan mengembangkan filsafat ilmu (Sain dan Pendidikan Sain)
7. Sikap yang menunjang pengembangan filsafat ilmu (Sain dan Pendidikan Sain)
8. Pengetahuan aspek pengembangan filsafat ilmu (Sain dan Pendidikan Sain)
9. Keterampilan pengembangan filsafat ilmu (Sain dan Pendidikan Sain)
10. Pengalaman pengembangan filsafat ilmu (Sain dan Pendidikan Sain)

XIII. Kegiatan Perkuliahan

Perte	Kompetensi Dasar	Materi Pokok	Strategi	Sumber

muan ke	Mahasiswa:		Perkuliahan	Bahan
1.	Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap persoalan-persoalan pokok dalam Pengembangan Ilmu(Sain dan Pendidikan Sain)	15. Persoalan-persoalan Pokok dalam Pengembangan Ilmu (Sain dan Pendidikan Sain)	Ekspositori, diskusi, presentasi, refleksi	Pilih yang sesuai
2.	Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap karakteristik ilmu(Sain dan Pendidikan Sain)	16. Karakteristik Ilmu (Sain dan Pendidikan Sain)	Ekspositori, diskusi, presentasi, refleksi	Pilih yang sesuai
3.	Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap obyek ilmu(Sain dan Pendidikan Sain)	17. Obyek dan Metode Ilmu (Sain dan Pendidikan Sain)	Ekspositori, diskusi, presentasi, refleksi	Pilih yang sesuai
4.	Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap alat pengembangan ilmu(Sain dan Pendidikan Sain)	18. Alat Pengembangan Ilmu (Sain dan Pendidikan Sain)	Ekspositori, diskusi, presentasi, refleksi	Pilih yang sesuai
	Ujian Sisipan I			
5.	Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap sejarah perkembangan ilmu(Sain dan Pendidikan Sain)	19. Sejarah Perkembangan Ilmu (Sain dan Pendidikan Sain)	Ekspositori, diskusi, presentasi, refleksi	Pilih yang sesuai
6.	Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap pre-asumsi dan asumsi dasar pengembangan ilmu(Sain dan Pendidikan Sain)	20. Pre-Asumsi dan Asumsi Dasar Pengembangan Ilmu (Sain dan Pendidikan Sain)	Ekspositori, diskusi, presentasi, refleksi	Pilih yang sesuai
7.	Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap Sumber-sumber dan	21. Sumber-sumber dan Batas-batas Pengembangan Ilmu (Sain dan Pendidikan	Ekspositori, diskusi, presentasi, refleksi	Pilih yang sesuai

	batas-batas pengembangan ilmu(Sain dan Pendidikan Sain)	Sain)		
8.	Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap pembenaran ilmu(Sain dan Pendidikan Sain)	22. Pembenaran dan Prinsip-prinsip Pengembangan Ilmu (Sain dan Pendidikan Sain)	Ekspositori, diskusi, presentasi, refleksi	Pilih yang sesuai
	Sisipan II			
9.	Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap ontologi ilmu(Sain dan Pendidikan Sain)	23. Ontologi Ilmu (Sain dan Pendidikan Sain)	Ekspositori, diskusi, presentasi, refleksi	Pilih yang sesuai
10.	Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap aliran ilmu(Sain dan Pendidikan Sain)	24. Berbagai Aliran Pengembangan Ilmu (Sain dan Pendidikan Sain)	Ekspositori, diskusi, presentasi, refleksi	Pilih yang sesuai
11.	Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap epistemologi ilmu (Sain dan Pendidikan Sain)	25. Epistemologi Ilmu (Sain dan Pendidikan Sain)	Ekspositori, diskusi, presentasi, refleksi	Pilih yang sesuai
12.	Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap aksiologi ilmu(Sain dan Pendidikan Sain)	26. Aksiologi Ilmu (Sain dan Pendidikan Sain)	Ekspositori, diskusi, presentasi, refleksi	Pilih yang sesuai
13.	Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap filsafat ilmu (Sain)	27. Filsafat Sain	Ekspositori, diskusi, presentasi, refleksi	Pilih yang sesuai
14.	Mengembangkan tesis, anti-tesis dan melakukan sintesis terhadap filsafat ilmu (Pendidikan Sain)	28. Filsafat Pendidikan Sain	Ekspositori, diskusi, presentasi, refleksi	Pilih yang sesuai
	UJIAN AKHIR SEMESTER			